A Theoretical Analysis of Public Funding for Research*

Gianni De Fraja†
University of Leicester
Università di Roma "Tor Vergata"
and C.E.P.R.

January 25, 2012

Abstract

This paper studies the government policy of funding for scientific research. A government agency needs to distribute funds among different research institutions and allocate them between basic and applied research. At the second best policy, the government uses basic research to induce the most productive institutions to carry out more applied research than they would choose if they had a fixed budget: the government information disadvantage causes too much concentration of research relative to the first best. With regards to mechanisms used in practice, the paper provides theoretical support for a dual channel funding mechanism, but not for full economic costing.

JEL Numbers: O38, H42, D82
Keywords: Basic and applied research, R&D, Scientific advances.

*I would like to thank Vincenzo Denicolò, Claudio Mezzetti, John Moore, Ludovic Renou and József Sákovics for helpful comments. Earlier versions were presented at the "Economics of science: Where do we stand?", conference, Observatoire des Sciences et Techniques, Paris, 4-5 April 2011, in Leicester, Edinburgh, and St. Andrews.

†University of Leicester, Department of Economics Leicester, LE1 7RH, UK, Università di Roma "Tor Vergata", Dip. SEFEMEQ, Via Columbia 2, I-00133 Rome, Italy, and C.E.P.R., 90-98 Goswell Street, London EC1V 7DB, UK; email: defraja@le.ac.uk.

1 Introduction

In 2008 in the OECD countries, government expenditure on scientific research amounted to around 0.8% of GDP, ranging from 0.2% in Mexico to 1.1% in Spain (OECD 2009). These large sums of taxpayers' money are channelled in many different ways: dedicated research centres, funding for public or private universities, subsidies to firms and private non-profit organisations among others. Also varied is the link between the funds provided and their destination: some funding is linked to specific research projects, some is distributed in consideration of past achievement, and some is simply awarded to institutions to spend as they see fit.

This variety raises immediate efficiency questions. Is the multiplicity of manners in which these funds are assigned a good thing? How should the total funding be distributed across different institutions? Would it be possible to re-allocate funding from one spending method to another and improve its impact on society? Should the funding agency be concerned with the nature, basic or applied, of the research carried out by the institutions which it funds? This paper provides a theoretical framework to address these questions: the aim here is to lay the foundations for a theory of the optimal public research spending.

My approach is microeconomic: I leave the macroeconomic aspect of total spending in the background, and concentrate instead on *balance between basic and applied* research and on the *distribution of funding among different research institutions.* The answers to these two questions, which are separate in the first best, turn out to be interlinked due to the information disadvantage suffered by the government. I make the plausible assumptions that research institutions differ in their characteristics and that the government's and the institutions' preferences regarding the balance between basic and applied research are not perfectly aligned. The government wishes to allocate its scarce resources to the institutions where they are most productive, and at the same time to ensure the "right" balance between basic and applied research. Institutions care about the "prestige" value of their research. Misalignment in preferences and differences among institutions create a non-trivial optimisation problem. As my analysis shows, the balance between basic and applied research and the allocation across institutions interact: the government uses *basic* research funding – more precisely, funding that the recipient institutions will choose to devote to basic research –, as a reward to induce more productive institutions to do more *applied* research. This is a suboptimal, second best policy: there is a distortion relative to the efficient allocation where the social marginal benefit of funding is constant across institutions: in words, research, both basic and applied, is inefficiently concentrated in the most efficient institutions.

The relative role of applied and basic research, at the centre of my study, requires their specific characteristics to be carefully identified and modelled accurately. The distinguishing feature I posit in this paper is that *it is harder to observe the benefits of basic research than it is to observe the benefits of applied research*. This idea matches closely the differences between basic and applied research highlighted in the existing literature. Typically, basic research (or, Strandburg (2005), fundamental, pure, curiosity-driven, upstream, unpredictable) is seen as driven by scientists' curiosity, its aim to acquire knowledge for knowledge's sake; in contrast, applied research is designed to solve practical problems. When it is measured, for example by the US National Science Foundation, research expenditure is classified precisely according to this criterion: "basic research is defined as systematic study directed toward fuller knowledge or understanding of the fundamental aspects of phenomena and of observable facts without specific applications towards processes or products in mind." Conversely, "applied research is defined as systematic study to gain knowledge or understanding necessary to determine the means by which a recognized and specific need may be met" (NSB 2008, p 7). Thus, simplifying only slightly, I posit that while all research is uncertain, an applied research project is one for which it is possible to specify the benefits in advance, and so to say *ex-post* whether or not it succeeded and realised these benefits. On the contrary, for basic research it is unknown in advance *where* a positive effect will emerge, if it does: therefore what the benefits of a basic research project might possibly be cannot by definition even be described *ex-ante*, and so the question of whether they have materialised is simply meaningless.[1] Formally, the model I propose

[1]Nelson (1959 pp 301–2) gives several examples of basic research projects pursued as an end in themselves, which unexpectedly assists the solution of an apparently completely unrelated applied research problem. Among the more recent ones, Moody (1995) describes in

is one where applied research is contractible, basic research is not.

Like some of the existing models (for example, Evenson and Kislev 1976, or more recently Aghion et al. 2008), I posit a hierarchical link between basic and applied research: the former precedes and provides the foundation to the latter. Specifically, I assume that applied research has a direct impact on the nation's income, whereas basic research has a direct impact only on the cost of carrying out applied research: applied research becomes easier, cheaper, more likely to succeed, and so on, when the body of basic research available to society is larger. I capture the unpredictability of the nature of the benefits arising from basic research with the assumption of a completely diffuse link between pure and applied research: in expectation, each applied research project is helped equally by every basic research project, implying that what matter for applied research is only total amount of basic research undertaken in society.

Having first shown, in Proposition 3, how information constraints force the government to use basic research as a reward to the more productive institutions to induce them to perform more applied research, and the consequent inefficiencies arising from difference in marginal net social return in different institutions, and between basic and applied research, the paper next explains how the optimal funding can be implemented in practice. I show that the dual funding system suggests itself naturally: in the optimal policy I derive, all institutions receive an identical "block grant", subject to a threshold level of applied research. More efficient institutions, which can do this minimum amount of applied research spending less than the "block grant" can therefore use their savings to engage in basic research. Additional funding is made avail-

detail the numerous strands of basic research which allowed the creation of the ubiquitous CD. A GPS navigation system would be far too inaccurate to be of any practical use without corrections of gravitational effects central to the theory of relativity (Haustein 2009). The abstract mathematical problem of covering a surface with symmetric tiles lies at the foundation of our understanding and exploitation of superconductors (Edelson 1992). Gauss's investigation into the distribution of prime numbers has led, with the contributions of some of the best mathematical minds over the course of two centuries, to the possibility of unbreakable cryptographic codes, without which e-commerce would not be possible (du Satoy 2003). Table 3 in Gersbach et al (2009) has a longer and more systematic list. An empirical investigation of the link between basic research conducted in universities and commercial applications of the applied research it generated is in Jensen and Thursby (2001).

able to institutions through a second funding channel. To receive these funds, an institution must carry out additional applied research: this corresponds to research grant funded applied research. Interestingly, the additional funding is lower than the cost of the additional applied research to be carried out: institutions need to "co-fund" any further applied research they wish to carry out. This is in contrast to the "cost-plus" approach favoured by funding agencies in the UK and elsewhere, which typically award research grants covering not only the full marginal cost but also a share of the institution's fixed costs.

The paper is organised as follows. Section 2 presents the model, and Section 3 the results. Section 4 shows how the policy can be implemented in practice; some additional remarks are in Section 5, and Section 6 concludes. Mathematical proofs are in the Appendix.

2 The model

2.1 Research institutions.

I model the publicly funded research sector of an economy. There is a continuum of institutions – private or public – with the potential to do research; their number is normalised to 1 without loss of generality. Institutions differ in their ability to spend public research funding productively. This ability is measured by an exogenously given parameter $\theta \in \left[\underline{\theta}, \bar{\theta}\right] \subseteq \mathbb{R}$. The distribution of θ in the sector is described by a differentiable function $F(\theta)$, with density $f(\theta) = F'(\theta) > 0$, for $\theta \in \left(\underline{\theta}, \bar{\theta}\right)$, and monotonic hazard rate, $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) > 0$. The most natural interpretation for θ is the skill of an institution's scientists,[2] but it can also encompass the institution's ability to supplement government funding with funds from private sources. These may include for example income from endowments, or, for universities, generated from students' tuition

[2] In the simplest model, each institution is randomly assigned its staff. In reality, of course, institutions compete for staff, which would make θ endogenous. In a fully developed model of the academic labour market, one would need to take into account the fact that researchers prefer to join high quality institutions, and so competition among institutions might not be based exclusively on salaries. Palomino and Sákovics (2004) is a model which combines competition among institutions (sports leagues in their paper), with externalities among its members (the individual teams).

fees, even though I do not model explicitly any technological complementarity or financial cross-subsidisation between teaching and research (see for example De Fraja and Valbonesi 2012). Another example is the possible commercial exploitation of research. Two institutions which differ in this respect, for example because of their contacts with industry, of the effectiveness of their technology transfer office or marketing department, will be characterised by different values of θ.

2.2 Private benefits and cost of research.

Research bestows prestige, and a research institution's objective is the maximisation of the total amount of research it carries out, r:

$$r(\theta) = a(\theta) + b(\theta), \tag{1}$$

where the link is made explicit between an institution's type, θ, and the amount of applied, $a(\theta)$, and basic, $b(\theta)$, research it carries out. The additive form in (1) simply implies that the marginal rate of substitution between applied and basic research is constant. Any preference that institutions might have is normalised away, and the substantive part of (1) is that institutional preferences between basic and applied research vary neither with their type θ, nor with the amount of basic and applied research they do.

Institutions do not "sell" their research directly to users, but use government funding to pay for it. Because of the government's funding policy, basic and applied research differ in their consequences for the institution's overall funding. They also differ in their effect on the institution's total cost, to which I turn next.

In general terms, a type θ research institution's cost of carrying out the amount a of applied research and the amount b of basic research can be written as[3]

$$\hat{c}(a, b, \theta, B). \tag{2}$$

[3] This is similar to Gersbach et al (2010), who posit that *aggregate* amount of basic research undertaken in society is a parameter of the function which gives the probability of a successful innovation in each of the continuum of industries where research is undertaken.

B is the *total* amount of basic research carried out in the economy:

$$B = \int_{\underline{\theta}}^{\bar{\theta}} b(\theta) f(\theta) \, d\theta, \tag{3}$$

where $b(\theta)$ is the average amount of basic research carried out in the institutions with productivity θ. In (2), total cost $\hat{c}(\cdot)$ increases with a, b, and θ, and decreases with B. This captures the externality bestowed on society by basic research: basic research reduces the cost of applied research, and it is the latter that has direct beneficial effects, as posited below, in Section 2.3.

A stylised model of the technology of research production suggests plausible restrictions on the shape of the cost function (2). Suppose that research is carried out by individual scientists employed by institutions. Scientists are motivated by the desire to gain the admiration of the academic community, for example by publishing in prestigious journals, by winning prizes and awards and so on. Institutions have the same motivation, but also a budget to allocate to the many potential projects as effectively as possible. They see the various research proposals from scientists they employ or consider appointing, and choose which to pursue: they hire, tenure, or reduce the teaching time of the scientists proposing a given project, authorise the lab costs necessary, and so on.

Suppose next that institutions can assess *applied* research projects, at least in expectation. Given the model sketched above, a first natural assumption is $c_{aa}(\cdot) > 0$: institutions see all the proposed projects, and choose which to carry out in order of "value for money", the cheapest first and so on, making each additional project dearer than the previous ones. Not so for *basic* research. Because basic research projects are impossible to evaluate ex-ante and so impossible to compare, institutions simply value them at the average value, and approve them sequentially (or according to some other criterion, also, necessarily, unrelated to their true value) until the desired total number is reached. Formally this implies $\hat{c}_{bb}(\cdot) = 0$. Next I posit $\hat{c}_{ab}(\cdot) = 0$. This is justified by the complete unpredictability of the beneficial effect of basic research: each applied research benefits equally from any basic research project, via the aggregate amount B, and so there is no complementarity between any given basic and any given applied research project, and in particular no complementarity between basic and applied research *within* an institution. $\hat{c}_{ab}(\cdot) = \hat{c}_{bb}(\cdot) = 0$

allows me to write the cost function (2) as $\hat{c}(a, b, \theta, B) = c(a, \theta, B) + bc^b(\theta, B)$ for some functions c and c^b.

This can be further simplified with the assumption that $c^b_\theta(\theta, B) = c^b_B(\theta, B) = 0$: neither θ nor B make it easier to assess the value of a basic research project. This might at first sight appear unrealistic; it is, however, in the spirit of the model of the production function for research I sketched above, whereby an institution finances a projects if it assesses it to have value exceeding a certain threshold. Consider the role of θ and B in an institution's choice of *applied* research projects. The nature of the uncertainty in applied research can be captured by the assumption that institutions observe the true value of a project with an error, and the error is larger when θ is higher and when B is lower. This implies $\hat{c}_{a\theta}(a, b, \theta, B) > 0$ and $\hat{c}_{aB}(a, b, \theta, B) \leq 0$.[4] However given my assumption that institutions are unable to assess at all the expected value of a basic research project *ex-ante* (or *ex-post* within a reasonable time interval), this argument does not apply to basic research. If all projects are assessed by the institution at the average value of a basic research project, irrespective of the value of θ and B, then $\hat{c}_{a\theta}(a, b, \theta, B) = \hat{c}_{aB}(a, b, \theta, B) = 0$. Therefore, the term $c^b(\theta, B)$ in the cost function given at the end of the previous paragraph

[4] This can be put formally. Suppose that an applied research project is characterised by a parameter $v \in \mathbb{R}$, and let the probability of success of a project of type v be $\varphi(v)$, $\varphi : \mathbb{R} \longrightarrow [0, 1]$ an increasing function. v is observed by the institution with an error: given a project of true type v, the institution observes $v + \varepsilon$, where ε is distributed, independently of v, with mean zero and variance $\sigma(\theta, B)$, with $\sigma_\theta(\cdot) > 0$: a lower θ institution commit smaller errors, and $\sigma_B(\cdot) < 0$: if the aggregate amount of basic research B is higher, then errors are smaller for everyone. Given this, to minimise the cost of achieving a number a of successful applied project, the institution will only select projects with a cut-off observed value above v^a, where

$$a = \int_{v^a}^{+\infty} \int_{-\infty}^{+\infty} \varphi(v) f^\varepsilon(\hat{v} - v) dv f^v(\hat{v}) d\hat{v},$$

where f^v and f^ε are the densities of v and ε respectively. The overall cost of obtaining these a successful projects, assuming that each projects has the same cost, is (proportional to) the number of projects whose observed value is v^a or above:

$$\int_{v^a}^{+\infty} \int_{-\infty}^{+\infty} f^\varepsilon(\hat{v} - v) f^v(v) dv d\hat{v}.$$

Cost is therefore lower when the variance $\sigma(\theta, B)$ is lower.

is constant and can be normalised to 1; the cost function (2) simplifies to

$$\hat{c}(a, b, \theta, B) = c(a, \theta, B) + b. \tag{4}$$

The simplification in (4) has also a useful implication for the interpretation of the results of the paper. I show below in Proposition 3 that, in conditions of imperfect information, low θ institutions, which are more efficient in carrying out *applied* research, do more *basic* research. This is not the case with symmetric information. Because $c_\theta^b(\theta, B) = 0$, it is clear that this is not because they are also better at basic research, but a consequence of a different mechanism: the fact that the funding agency's information disadvantage forces it to offer funding that institutions can use to pay for basic research in order to induce those institutions that are efficient at applied research to do more of it than they would like. In other words, the stylised fact that better institutions do more basic research is *not* a consequence of their superior ability to do *basic* research, but of their superior ability to do *applied* research.

These restrictions on the cost function are collected formally in Assumption 1.

Assumption 1 *The cost function of a type θ institution is given by (4). For every $a, B \geqslant 0$, for every $\theta \in [\underline{\theta}, \bar{\theta}]$, the function $c(a, \theta, B)$ satisfies:*

1. $c_a(\cdot) > 0$, $c_\theta(\cdot) > 0$, $c_B(\cdot) < 0$, $c(0, \theta, B) = 0$.
2. $c_{aa}(\cdot) > 0$, $c_{BB}(\cdot) > 0$, $c_{a\theta}(\cdot) > 0$, $c_{Ba}(\cdot) \leqslant 0$.
3. $-c_B(a, \theta, 0) > 1$ *for every* $a \geqslant 0$, *for every* $\theta \in [\underline{\theta}, \bar{\theta}]$.

To recap, Assumption 1.1 simply defines θ as a measure of cost, captures the externality created by B, and rules out fixed costs. The second set of hypotheses are natural decreasing returns to scale assumptions ($c_{aa}(\cdot) > 0$ and $c_{BB}(\cdot) > 0$), and that a lower θ and more basic research decrease the marginal cost, as well as the total cost. Assumption 1.3 avoids unrewarding corner solutions by ensuring that if there is no basic research in society then a very small amount reduces the cost of research by more than it costs.

2.3 Social benefits and cost of research

Basic research affects welfare only though its effect on the productivity of individual institutions' cost of carrying out applied research. Instead, applied research affects *directly* national income, which I define as

$$Y(A), \tag{5}$$

with $Y'(A) > 0$, $Y''(A) \leqslant 0$.

$$A = \int_{\underline{\theta}}^{\bar{\theta}} a(\theta) f(\theta) \, d\theta, \tag{6}$$

denotes the expected and actual *total* amount of successful applied research, where $a(\theta)$ denotes the average amount of applied research carried out in the institutions with productivity θ. A is therefore a standard Solow residual.

As with basic research, the externality does not create the appropriability problems which beset R&D activities carried out in profit maximising firms, well-understood by the literature since at least Arrow (1962). This is both because all effects of research are internal to the government, which funds research,[5] and because individuals and institutions doing research are not concerned with its monetary appropriability: their reward is the *production* of knowledge, not its financial exploitation, as has long been recognised (see Stephan 1996 for a comprehensive review). Therefore the value of a given project a is the same for a fully appropriable one, for example the development of a new therapy by a private profit-making pharmaceutical company receiving a government subsidy to research, or a research centre or a university selling a patent through a TTO,[6] or one with more diffuse benefits, such as an improvement in communication technology, which benefit all firms and consumers.

The government's objective function is the total national income, reduced by the cost of the taxes necessary to fund the research sector, which includes a

[5] Of course in an international context, some of the benefits determined by the expenditure of one country's taxpayers' money do accrue to different countries. This can be captured by re-interpreting some of the parameters that measure the benefit of research or the shadow cost of public funds, λ and k in (7), to take this international spillover into account.

[6] The analysis of the role and effects of Technology Transfers Offices, outside the scope of this paper, can be found for example, in Macho-Stadler et al (2007) and in the references reported there.

distortionary component, plus the non-monetary benefit of research (prestige, etc.). Formally, the government's payoff function is

$$k(A+B)+Y(A)-(1+\lambda)T, \tag{7}$$

where $k \in \mathbb{R}$ is the weight of the non-monetary benefit of research,[7] $\lambda > 0$ the shadow cost of public funds, and T the total funding to research.

The government chooses its research funding policy: this takes the shape of a link between the amount of applied research carried out by an institution and the funding provided by the government to that institution: formally, a policy is a function $C(a)$, where C is the total payment to an institution that carries out amount a of research. Factoring in institutions' optimisation problems, a policy $C(a)$ can be written as a pair of functions, $\{b(\theta), a(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$, the amount of basic and applied research in a type θ institution, where $b(\theta) = C(a(\theta)) - c(a(\theta), \theta, B)$, or equivalently as $\{r(\theta), a(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$, the amount of total and applied research in a type θ institution.

Given a research policy, the total tax needed to fund it is simply $T = \int_{\underline{\theta}}^{\bar{\theta}} C(a(\theta)) f(\theta) d\theta$, and, in view of (1), (3) can be replaced by:

$$B = \int_{\underline{\theta}}^{\bar{\theta}} [r(\theta) - a(\theta)] f(\theta) d\theta. \tag{8}$$

The following ensures that applied research is sufficiently important.

Assumption 2 *For every* $A \geqslant 0$, $\frac{Y'(A)+k}{1+\lambda} > 1$.

That is, the marginal social benefit of applied research exceeds the private benefit,[8] implying an externality in applied research. Since the private benefit of applied research equals the private cost of basic research, and the latter equals the social cost of basic research, Assumption 2 implies that the social benefit of *applied* research exceeds the social marginal cost of *basic* research.

[7] k could be negative, indicating a "philistine" government: my results imply that even such a government would tolerate, indeed *fund* basic research.

[8] Strictly speaking, the private benefit in the absence of government incentives.

3 Results

3.1 Preliminaries

The viewpoint of this paper is normative: I study how the government should distribute the research budget across institutions and direct institutions' choice of the balance between applied and basic research to maximise its objective function (7) subject to information constraints, discussed in detail below. Before I present the results, it is convenient to define the amount of applied research which equates the marginal return on applied and basic research.

Definition 1 *$a^*(\theta; B)$ is value of a which solves*

$$c_a(a, \theta, B) = 1. \tag{9}$$

That is, $a^*(\theta; B)$ is the amount of applied research which maximises type θ institution's objective function when the aggregate amount of basic research is B, provided the funding available to the institution is large enough not to constrain applied research; this can be defined as the *individually efficient expenditure on applied research*. Note that $\frac{\partial a^*(\cdot)}{\partial \theta} = -\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} < 0$ and $\frac{\partial a^*(\cdot)}{\partial B} = -\frac{c_{aB}(\cdot)}{c_{aa}(\cdot)} \geqslant 0$. That is, more efficient institutions have a higher individually efficient expenditure on applied research, and an increase in the level of basic research increases the individually efficient expenditure on applied research for all universities. This "crowding in" of basic research (Malla and Gray 2005, p 434) seems natural.

3.2 Perfect information

The first proposition gives the benchmark case in which the government has complete information. Let $a_1(\theta)$, A_1 and B_1 be defined by:

$$c_a(a_1(\theta), \theta, B_1) = \frac{Y'(A_1) + k}{1 + \lambda}, \tag{10}$$

$$A_1 = \int_{\underline{\theta}}^{\bar{\theta}} a_1(\theta) f(\theta)\, d\theta, \tag{11}$$

$$\frac{k}{1 + \lambda} = \int_{\underline{\theta}}^{\bar{\theta}} c_B(a_1(\theta), \theta, B_1) f(\theta)\, d\theta + 1. \tag{12}$$

By Assumption 2, $\frac{Y'(A_1)+k}{1+\lambda} > 1$, and so $a_1(\theta) > a^*(\theta; B_1)$.

Proposition 1 *If the government could observe perfectly the productivity of each institution and the amount of applied and basic research each institution carries out, it would choose: $a_1(\theta)$, and any function $r(\theta) \geqslant a_1(\theta)$ such that $\int_{\underline{\theta}}^{\bar{\theta}} r(\theta) f(\theta) d\theta = A_1 + B_1$.*

The proofs of all results are relegated to the Appendix. Notice that, since $-c_B(a_1(\theta), \theta, 0) > 1 > 1 - \frac{k}{1+\lambda}$, by virtue of Assumption 1.3, and $c_{BB}(\cdot) > 0$ in Assumption 1.2, then B_1 determined in (12) is strictly positive.

Proposition 1 is a straightforward first best result: with perfect information, the government simply asks each institution to carry out a certain amount of applied research. Since $\frac{Y'(A_1)+k}{1+\lambda} > 1$, this amount exceeds $a^*(\theta; B_1)$, that is, it exceeds what each institution would choose if it were simply given a budget to spend as it pleases. This is natural: the government internalises the benefits of applied research, and so it derives a larger benefit from it than individual institutions do. Note that (10) implies that the marginal cost of doing applied research is the same in every institution. This is efficient; if it were not the case, the government could transfer research from one institution to another and reduce the overall cost of applied research.

Notice that more efficient universities do more applied research: $a_1'(\theta) = -\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} < 0$. They are better at it, so this is natural. Equally natural is the fact that the distribution of basic research across universities is a matter of indifference.[9] Since all institutions are equally productive at basic research, the government determines the total amount of basic research in (12) and then distributes it in any feasible way, that is in any way such that $b(\theta) \geqslant 0$ for every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$. Lastly, note that an increase in k and a reduction in λ increase $a_1(\theta)$ for every $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, thus increasing A_1 and B_1.

I consider next a lower information requirement on the government's part.

[9] With a general cost function (2) instead of (4), $a^*(\theta; B)$ would be defined not by (9), but by the solution in a to $c_a(a, \theta, B) = c^b(\theta, B)$; in this case the government would allocate basic research to the lowest cost institutions only, or, assuming also decreasing returns to scale in basic research, in such a way to equalise the marginal cost of basic research. The other qualitative features of the analysis would not change.

Corollary 1 *If the government can observe each institution's type and the amount of applied research it carries out and, but cannot observe the amount of basic research, then it would choose* $a_1(\theta)$ *and any function*

$$C(a_1(\theta)) \geqslant c(a_1(\theta), \theta, B_1) \tag{13}$$

such that $\int_{\underline{\theta}}^{\bar{\theta}} [C(a_1(\theta)) - c(a_1(\theta), \theta, B_1)] f(\theta) d\theta = B_1$.

That is, the optimal policy when the government is unable to observe basic research is exactly as in Proposition 1. The reason is straightforward: the government wants each institution to carry out the appropriate amount of applied research, and offers individualised contracts to each institution with this requirement. These contracts require an amount of applied research higher than what the institution would do on its own ($a_1(\theta) > a^*(\theta; B_1)$): they are therefore incentive compatible. At least some institutions are offered more funding than they need for the applied research they are asked to carry out: for them the LHS of (13) exceeds the RHS. These institutions use any excess funding to pay for their basic research, which, as before, is distributed in any feasible way.

The policy derived here cannot however be implemented if the government cannot observe each institution's productivity parameter, θ. The reason is that Proposition 1 and Corollary 1 require each institution to choose a combination of basic and applied research effort such that applied research has a higher marginal cost than basic research. Hence, if institutions were simply offered funding $C(a_1(\theta))$ and asked to carry out $a_1(\theta)$ applied research, they would have an incentive to claim to have a higher θ than they have. By doing so, they would receive less funding but nevertheless be able to increase the total amount of research they do with their funding, as they would be able to switch away from the more costly applied research and do more basic research. Formally, presented with the link between a funding level menu $C(a_1(\theta))$, a type θ institution would claim to be of type $\min\left\{a_1^{-1}(a^*(\theta; B)), \bar{\theta}\right\}$. In this way, its marginal cost of doing applied research is as near as possible to 1, its marginal cost of basic research.

3.3 Information asymmetry.

I now consider the more realistic information structure where the government can observe neither θ, an institution's productivity, nor b, the amount of basic research it does. Instead, the government can observe, and an external adjudicator can verify, whether an agreed level of applied research a has been carried out. In practice, and in line with the toy model sketched above, *ex-ante* uncertainty about research plays a limited role: although each project is uncertain, an institution carrying out a portfolio of applied research projects will roughly know how many will succeed. An aggregate payment linked to the number of successful applied research projects, within a reasonably short time horizon, is therefore a feasible contract. In sum, applied research is observable and contractible, basic research is not.[10]

3.4 Incentive Compatibility

In this subsection, I determine the constraints imposed by the information disadvantage of the government. I describe them via the standard revelation approach, that is supposing that the government asks each institution to report its own type, having committed to a policy as a function of the reported type; by the revelation principle, the government cannot improve on the payoff it can obtain by restricting its choices to policies that satisfy the incentive compatibility constraint, that is the property that no institution has an incentive to misreport its type. This constraint is derived next. Recall that a policy $\{C(a(\theta)), a(\theta)\}_{\theta\in[\underline{\theta},\bar{\theta}]}$ can be written as $\{r(\theta), a(\theta)\}_{\theta\in[\underline{\theta},\bar{\theta}]}$, with $r(\theta)$ and $a(\theta)$ the total and the applied research required of institution of type θ, $\theta \in [\underline{\theta}, \bar{\theta}]$.

[10] This is recognised in peer review based formal evaluation mechanisms, which are intended to assess the research effort of institutions, both in applied and in basic research. For example, the version of the exercise carried out in 2014 in the UK, known as REF, has two measures of output, academic publications, judged solely on their academic merit, irrespective of their applied or basic nature, and *impact* of research on society: this needs to quantify "all kinds of social, economic and cultural benefits and impacts beyond academia" (HEFCE 2011, p 4), and, given both the short time horizon and the exacting standard of the required link from research to benefits, it is arguably only applicable to applied research.

Proposition 2 *A policy* $\{r(\theta), a(\theta)\}_{\theta\in[\underline{\theta},\bar{\theta}]}$ *is feasible and incentive compatible if it satisfies, for* $\theta \in [\underline{\theta}, \bar{\theta}]$*:*

$$\dot{r}(\theta) = -c_\theta (a(\theta), \theta, B), \qquad r(\underline{\theta}) \text{ free; } r(\bar{\theta}) = 0, \tag{14}$$

$$\dot{a}(\theta) \leqslant 0, \tag{15}$$

$$a(\theta) - a^*(\theta; B) \geqslant 0, \tag{16}$$

$$r(\theta) - a(\theta) \geqslant 0. \tag{17}$$

(14) and (15) are standard, (16) follows from the requirement that cost be decreasing in θ: if not, an institution could pretend to be of a worse type, receive more funding and also be required to do less applied research. (17) is simply $b(\theta) \geqslant 0$.

3.5 The optimal funding policy

I am now in a position to present the government maximisation problem. This is the choice of a policy $\{r(\theta), a(\theta)\}$, which satisfies the constraints given in Proposition 2 and maximises the government objective function. As in Section 3.2, A and B, the aggregate amount of applied and basic research are best treated as parameters chosen by the government, subject of course to their respective definition constraints, (6) and (8). Requirement (16), that $a(\theta) > a^*(\theta; B)$, implies that the government might want to exclude some institutions. If it does so, it will exclude institutions with θ above a certain threshold. This threshold, $\theta_0 \in [\underline{\theta}, \bar{\theta}]$, is itself a choice variable.

The government's problem is therefore the following.

$$\max_{\substack{r(\theta),a(\theta),\\ A,B,\theta_0}} Y(A) + k(A+B) - (1+\lambda) \int_{\underline{\theta}}^{\theta_0} [c(a(\theta), \theta, B) + r(\theta) - a(\theta)] f(\theta) d\theta$$

$$\text{s.t.: (6), (8), (14), (15), (16), (17).} \tag{18}$$

I can now derive the optimal funding policy. This is based on three functions, $a^*(\theta; B)$ given above in Definition 1, and $a^K(\theta; B, \beta)$ and $r^*(\theta; B, \theta_0)$, defined next. For given parameters $B > 0$, $\beta \geqslant 0$, let $a^K(\theta; B, \beta)$ be the solution in a of

$$c_a(a, \theta, B) = \frac{Y'(A) + k}{1+\lambda} + \beta - \frac{\beta F(\theta)}{f(\theta)} c_{\theta a}(a, \theta, B). \tag{19}$$

The curve $a^K(\theta; B, \beta)$ is drawn in Figures 1 and 4, and is intuitively discussed following Corollary 2. Next, let $r^*(\theta; B, \theta_0)$ be the solution to the following differential equation:

$$\dot{r}(\theta) = -c_\theta(r(\theta), \theta, B), \qquad r(\theta_0) = a^*(\theta_0; B). \tag{20}$$

That is, for given B, $r^*(\theta; B, \theta_0)$ is the function $r(\theta)$ which satisfies the incentive compatibility constraint "shifted" so as to intersect $a^*(\theta; B)$ at $\theta = \theta_0$.

Assumption 3 *(i)* $c_\theta(\cdot) > \frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)}$; *(ii)* $c_\theta(\cdot) > \frac{c_{\theta\theta a}(\cdot)}{c_{\theta aa}(\cdot)} + \frac{c_{a\theta}(\cdot)}{c_{\theta aa}(\cdot)} \frac{\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{\frac{F(\theta)}{f(\theta)}}$, *(iii)* $c_{\theta aa}(\cdot) > 0$.

Loosely speaking, the first two statements in Assumption 3 restrict $c_\theta(\cdot)$ to be "large enough": cost differences among institutions, measured by the parameter θ, must be sufficient important. θ, the measure of the differences in productivity among institutions, is unobservable, and so the first two statements in Assumption 3 simply require that the information disadvantage of the government be sufficiently serious. It is this disadvantage that makes the analysis relevant: in the extreme case where all research institutions have the same cost, $c_\theta(\cdot) = 0$, the government's inability to observe their productivity is obviously not a problem. Of course, the considerable effort that funding agencies exert to ascertain the research potential of the research institutions they sustain does suggest strongly that these differences are indeed important in practice. The third statement in Assumption 3 is a regularity restriction.

Proposition 3 *Let Assumptions 1-3 hold. If problem (18) has a solution, then there exist* $\tilde{\theta}, \theta_K \in (\underline{\theta}, \theta_0]$ *with* $\underline{\theta} < \theta_K \leqslant \tilde{\theta} \leqslant \theta_0$ *such that:*

if $\theta \in [\underline{\theta}, \theta_K)$ *then* $a(\theta) > a^*(\theta; B)$ *and* $b(\theta) > 0$;
if $\theta \in \left[\theta_K, \tilde{\theta}\right)$ *then* $a(\theta) = a^*(\theta; B)$ *and* $b(\theta) > 0$;
if $\theta \in \left[\tilde{\theta}, \theta_0\right]$ *then* $a(\theta) = a^*(\theta; B)$ *and* $b(\theta) = 0$;
if $\theta \in \left[\theta_0, \overline{\theta}\right]$ *then* $a(\theta) = b(\theta) = 0$.

The following implication of Proposition 3 is worth stating formally, as it helps illustrate the optimal policy in a diagram.

Corollary 2 *Let Assumptions 1-3 hold. If problem (18) has a solution, then there exist* $B > 0$*,* $\theta_0 \in [\underline{\theta}, \overline{\theta}]$*, and* $\beta > 0$*, such that:*

$$a(\theta) = \min\left\{\max\left\{a^*(\theta; B), a^K(\theta; B, \beta)\right\}, r^*(\theta; B, \theta_0)\right\}$$
$$b(\theta) = \max\left\{r^*(\theta; B, \theta_0) - \max\left\{a^*(\theta; B), a^K(\theta; B, \beta)\right\}, 0\right\}$$

for $\theta \in [\underline{\theta}, \theta_0]$*, and* $a(\theta) = b(\theta) = 0$ *for* $\theta \in (\theta_0, \overline{\theta}]$*.*

The optimal policy described in Proposition 3 and Corollary 2 is depicted in Figure 1. In each panel of the Figure, the vertical axis measures the amount of research, and the horizontal axis an institution's productivity parameter, θ. Only institutions with θ below θ_0, the intersection of $r^*(\theta; B, \theta_0)$ and $a^*(\theta; B)$, receive any government funding. In each panel, there are three different curves. The solid thin line is the locus $r^*(\theta; B, \theta_0)$: by Proposition 2, it represents the total amount of research carried out by a type θ institution. The dotted line is locus $a^*(\theta; B)$, and the dashed line is $a^K(\theta; B, \beta)$. By Corollary 2, the amount of *applied* research each institution does is given by the higher of these two curves, if it is below $r^*(\theta; B, \theta_0)$, and otherwise by $r^*(\theta; B, \theta_0)$ itself. In the latter case, the institution does no basic research. In both panels, the solid thick curve is the amount of applied research carried out by a type θ institution, and the distance between the latter curve and $r^*(\theta; B, \theta_0)$, shaded in grey in the diagrams, is the amount of basic research done by a type θ institution.

The proof of Proposition 3 shows that β in (19) is positive. $(1 - \beta)$ is the multiplier associated with the distortion in (aggregate) basic research. It is 1 (and so $\beta = 0$) with symmetric information, as the comparison between (10) and (19) shows. Together with Assumption 2, this implies that the dashed line $a^K(\cdot)$ is above the dotted line $a^*(\cdot)$ in a right neighbourhood of $\underline{\theta}$: the most efficient institutions do more research than they would like.[11]

The panels of Figure 1 differ in the position of the curve $a^K(\theta; B, \beta)$. The latter can have three kinds of relationship with the two other relevant curves,

[11] In aggregate, the marginal cost of applied research is higher than the marginal cost of basic research. This is in line with the empirical evidence which suggests that basic research expenditure is a more important productivity determinant than applied research (e.g., Mansfield 1980; Link 1981; Griliches 1986). It follows from the optimal policy: if the government simply financed research without trying to direct it (for example through lump sum unconditional transfers), applied and basic research would have the same social benefit.

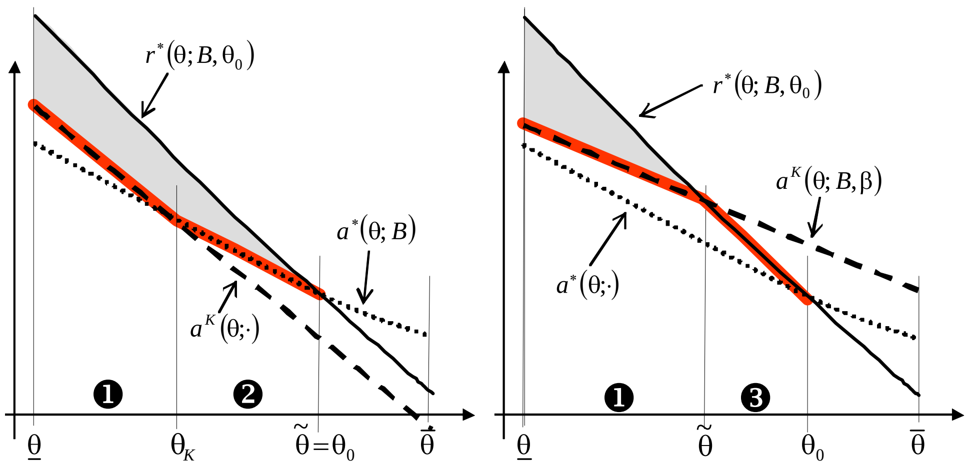

Figure 1: Applied and basic research. The second best policy.

indicated by the white numbers in a black disk, reflecting the three possible patterns of complementary slackness of the constraints in Problem (18). In region 1, both (16) and (17) are slack; in region 2, constraint (16) is binding. In region 3, (17) is binding. The conceptual difference between regions 1 and 2 is that research institutions in region 2 choose their "preferred" combination of applied and basic research, and those in region 1 are required/incentivised to do more than this amount. Research institutions in region 3 do only applied research.

The position of the curve $a^K(\theta; B, \beta)$ and hence which region a given institution type θ is in, is affected by four factors, as shown by (19): the direct effect of applied research on national income, $Y'(A)$; the "prestige" effect of research on the policy maker's payoff, k; the shadow cost of public funds λ; and finally, the endogenously determined overall effect of basic research on institutions' cost of doing applied research, β. The first three simply shift the dashed curve $a^K(\theta; B, \beta)$ up and down in a parallel fashion (leaving aside the indirect effect through B). Thus, other things equal, increases in k and in $Y'(A)$ and decreases in λ all increase the amount of applied research, and decrease the amount of basic research. The sign of the effect for k and λ follows from the fact that the cost of applied research is higher that that of basic research, therefore, if research becomes more desirable, higher k, or cheaper to fund, lower

λ, more of the "expensive" type, applied research, will be done, moving the optimal policy further away from the institutions preferences.

The fourth term β has a more complex role. It captures the distortionary effect of information asymmetry (and so, as (10) and (19) show, it is 0 with perfect information). Apparently in contrast to the "efficiency at the top" principle, when $\beta > 0$ the amount of applied research done by the most efficient institution is higher than the first best amount. The reason is that information asymmetry affects the total amount of basic research that it is optimal to carry out, and therefore the socially optimal amount of applied research that the most efficient institution should do. For higher values of θ, however this effect is reduced by the incentive effect: in order to induce institutions to self-select, it is necessary to prevent high types from pretending to be low type, and thus doing less applied research and devote funds to the cheaper basic research: reducing sufficiently the amount of applied research makes this less attractive for a good institution than for a bad one. Basic research is therefore used to provide incentives for institutions to reveal their type and commit to carry out applied research in excess of $a^*(\theta; B)$. The reduction in applied research for high cost institutions is taken to the extreme for $\theta > \theta_0$: they would do some research if the government could observe their type, but receive no funding when the government has imperfect information.

The next Section describes in detail how the link between applied research and total funding can be implemented in practice. Before, I consider a special case, which illustrates the role of the basic research externality.

Assumption 4 *The cost function $c(a, \theta, B)$ is additively separable in (a, θ) and B: there exist $\tilde{c}(a, \theta)$ and $\zeta(B)$ such that $c(a, \theta, B) = \tilde{c}(a, \theta) + \zeta(B)$ for every (a, θ, B).*

In words, B affects only the fixed cost of doing applied research.

Corollary 3 *If assumption 4 holds, then $c_a(a(\theta), \underline{\theta}, B) = 1 + \left(\frac{Y'(A)}{1+\lambda} + \zeta'(B) F(\theta_0)\right)$.*

Recall that for the curve $a^*(\theta; B)$, it is $c_a(a^*(\cdot), \underline{\theta}, B) = 1$. Therefore by how much applied research is pushed above the efficient level in the most productive institutions, that is by how much the "starting" point of the dashed curve $a^K(\cdot)$ exceeds that of the the curve $a^*(\theta; B)$, depends on the extent by which $\frac{Y'(A)}{1+\lambda}$, the marginal benefit of an increase in applied research, exceeds $\zeta'(B)\,F(\theta_0)$, the marginal cost of an increase in basic research.

4 Implementation

This section investigates how a central funding agency can implement in practice the optimal policy derived in Proposition 2. This agency is constrained by the fact that basic research is unobservable. All it can therefore do is to stipulate a link between the amount of applied research carried out and the total amount of funding an institution receives. Mathematically, this is a function $C(a)$, which gives the total funding as a function of the total amount of applied research carried out. Since there is a one-to-one relationship between θ and a, this is well defined.

The shape of this function depends on which of the three regions identified in Proposition 2 the optimal choice belongs to. To see this, consider an institution of type θ which, given the incentive compatible policy $\{r(\theta), a(\theta)\}_{\theta\in[\underline{\theta},\bar{\theta}]}$, chooses $r(\theta)$ and $a(\theta)$ and therefore it obtains total funding $C(a(\theta))$. In region 1, the total funding it receives is

$$C(a(\theta)) = c\left(a^K(\theta;B,\beta),\theta,B\right) + \left[r^*(\theta;B,\theta_0) - a^K(\theta;B,\beta)\right].$$

The first term is the cost of carrying out $a^K(\theta;B,\beta)$ amount of applied research and the terms in the square brackets the cost (and the amount) of basic research. For fixed B and β, let $\theta^K(a;B,\beta)$ be the inverse of the function $a^K(\theta;B,\beta)$: $\theta^K(a;B,\beta)$ is the value of θ such that $a^K(\theta;B,\beta) = a$. Consider now an institution which, faced with a schedule $C(a)$ chooses to carry out an amount a of applied research; if the policy is incentive compatible, it has type $\theta^K(a;B,\beta))$. The total funding it receives is given by:[12]

$$C(a) = c\left(a,\theta^K(a;B,\beta),B\right) + r^*\left(\theta^K(a;B,\beta);B,\theta_0\right) - a. \tag{21}$$

[12] Notice that, faced with (21), a type θ institution does indeed want to carry out precisely $a = a^K(\theta;B,\beta)$ applied research. To see this, note that it will solve

$$\max_{a\geqslant 0}\max\left\{a + [C(a) - c(a,\theta,B)], a^*(\theta;B)\right\},$$

where $C(a)$ is given by (21). The first order condition for the above is $c_a\left(a,\theta^K(a;B,\beta),B\right) = c_a(a,\theta,B)$.

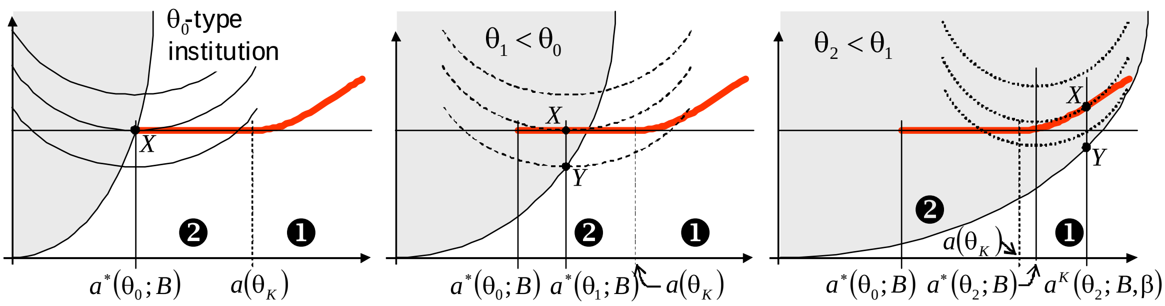

Figure 2: Implementation: The LHS of Figure 1.

Corollary 4 *If* $a(\theta) = \theta^K(a; B, \beta)$*, then* $C(a)$ *is increasing.* $C(a)$ *is convex if and only if* $\frac{\partial a^*(\theta;B)}{\partial\theta} > \frac{\partial a^K(\theta;B,\beta)}{\partial\theta}$.

Therefore $C(a)$ is convex if the relative slope of the dashed and dotted curves is as in the LHS of Figure 1, concave if it is as in the RHS.

The same procedure can be applied to derive the shape of $C(a)$ in the other regions. Begin with region 2. Here, let $\theta^*(a; B)$ be the inverse function of $a^*(\theta; B)$, so that total funding is given by:

$$C(a) = c(a, \theta^*(a, B), B) + r^*(\theta^*(a, B); B, \theta_0) - a. \tag{22}$$

Corollary 5 *If* $a(\theta) = a^*(\theta; B)$*, then* $C(a)$ *is constant.*

Finally region 3. With the same argument used for region 1, I show that the function is again increasing and convex.

Corollary 6 *If* $a(\theta) = r^*(\theta; B, \theta_0)$*, then* $C(a)$ *is increasing and convex. Moreover, at the boundary between region 1 and 3 the slope of* $C(a)$ *is increasing in* a.

Having determined the slope of the function $C(a)$, I show in Figure 2 how the funding agency can implement it in practice. The Figure shows a cartesian space with the amount of applied research on the horizontal axis, and the total funding on the vertical axis for three different institution types. The function $C(a)$, defined for $a \in \left[a^*(\theta_0; B), \max\left\{a^K(\underline{\theta}; B, \beta), a^*(\underline{\theta}; B)\right\}\right]$ is the thick solid line: points on this locus represent combinations of funding and applied research which the funding agency allows research institutions to choose from. Since the funding agency cannot tell types apart, it is the same in the three panels, and it is drawn for the case depicted in the LHS panel of Figure 1. Each diagram also shows the indifference curves, and, shaded, the "feasible set", the combinations of funding and the amount of applied research which a type θ institution is able carry out with that funding. The LHS, the middle and the RHS panel show these for the least productive active institution, for one with high cost, and for a very low cost institution, respectively. Consider first a type θ_0 institution, shown on the LHS panel. Its indifference curves are the solid thin lines;[13] its feasible set, the grey shaded area, is the set $\{(a,t) \in \mathbb{R}^+ | c(a,\theta_0,B) \leqslant t\}$. This institution has effectively no choice: only the point $(a^*(\theta_0;B), C(a^*(\theta_0;B)))$ marked by X in the LHS diagram, is both on the solid thick locus and in the "feasible set". Not so however for more productive research institutions: take type $\theta_1 \in (\theta_K, \theta_0)$, illustrated in the middle panel. Its indifference curves are the dashed lines, and its feasible set again the grey area, clearly bigger than a type θ_0's. It therefore has a genuine choice among the points in the grey area and on the thick solid line. The best among such points is $(a^*(\theta_1;B), C(a^*(\theta_1;B)))$, point X in the diagram. Notice that the required level of applied research, $a^*(\theta_1;B)$, will cost this institution only $c(a^*(\theta_0;B), \theta_0, B)$, the vertical height of point Y, which is less that $C(a^*(\theta_1;B))$. After it has paid for its applied research, it will spend its "leftover" funding on basic research, which has marginal cost of 1, rather than on more applied research, which, if pushed above $a^*(\theta_1;B)$, would have a marginal cost exceeding 1. A type θ_1 institution therefore carries out an amount of basic research measured by the vertical distance between points Y and X.

Finally consider a very efficient institution, one with cost parameter $\theta_2 < \theta_K$. Its efficient level of applied research is $a^*(\theta_2;B)$, the abscissa of the minima of the indifference curves in the RHS panel of Figure 2. This is the level it would choose if funding were constant. But the optimal policy is designed so that this institution does more than this amount: the funding agency offers increasing funding for research institutions which exceed their efficient level

[13] They all reach a minimum at $a = a^*(\theta_0;B)$, as can be seen by totally differentiating $a + t - c(a,\theta_0,B)$

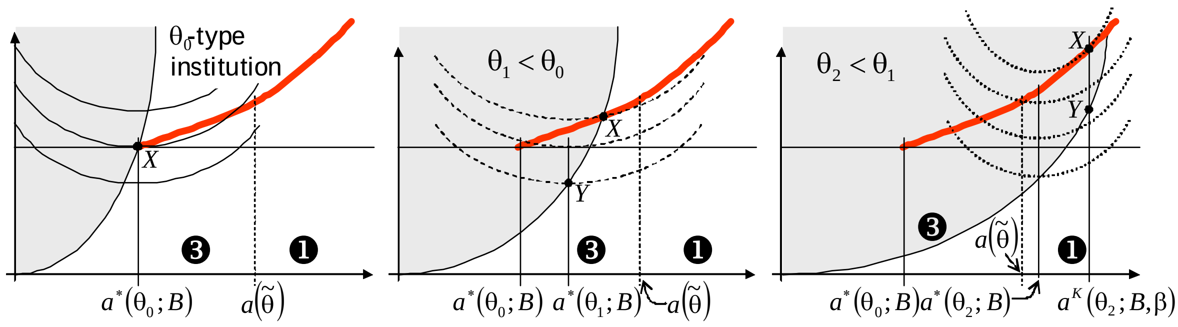

Figure 3: Implementation: The RHS of Figure 1.

of applied research. Faced with the solid thick schedule, a type θ_2 research institution chooses the combination that allows it to be on the highest possible indifference curve, namely tangency point X in the RHS panel of diagram.[14] This institution's cost to carry out the amount of applied research $a^K(\theta_2; B, \beta)$ is the ordinate of point Y, and so a type θ_2 institution spends the rest, measured by the distance between X and Y, on basic research.

When the relative position of the curves $a^*(\theta_0; B)$ and $a^K(\theta; B, \beta)$ is instead that shown in the RHS panel of Figure 1, the optimal funding can be implemented by the schedule illustrated in Figure 3. This differ from Figure 2 only in that the initial part of the schedule is also increasing. The RHS and the LHS panels are conceptually identical in Figures 3 and 2: a type θ_0 institution has no choice (LHS) and efficient institutions do more applied research than they would like (RHS), and have enough funding to do basic research. In the middle panel, in contrast, an institution of an intermediate θ is seen to spend all of its budget on applied research, to do more than its efficient level of applied research, and to have no funding left for basic research: in the picture,

[14]When the curve $C(a)$ is convex, as in Figure 2, then the tangency point is a local, and hence a global, maximum. To see this, note that, at the tangency point $(a_2, C(a_2))$, with $a_2 = a^K(\theta_2; B, \beta)$, the slope of the indifference curve is given by $c_a(a, \theta, B) - 1$. The slope of the funding schedule is given by (A16). In a neighbourhood of a_2, we have:

$$c_a(a_2+\varepsilon, \theta_2, B) - c_a\left(a_2+\varepsilon, \theta^K(a_2+\varepsilon; B, \beta), B\right) = c_{a\theta}(a_2+\varepsilon, \theta_3, B)\left(\theta_2 - \theta^K(a_2+\varepsilon; B, \beta)\right).$$

For $\varepsilon > 0$ (resp $\varepsilon < 0$), the above is positive (negative), as $a^K(\cdot)$ is decreasing and so $\theta^K(\cdot)$ is too. Clearly if the curve $C(a)$ is concave, then the tangency point is a maximum.

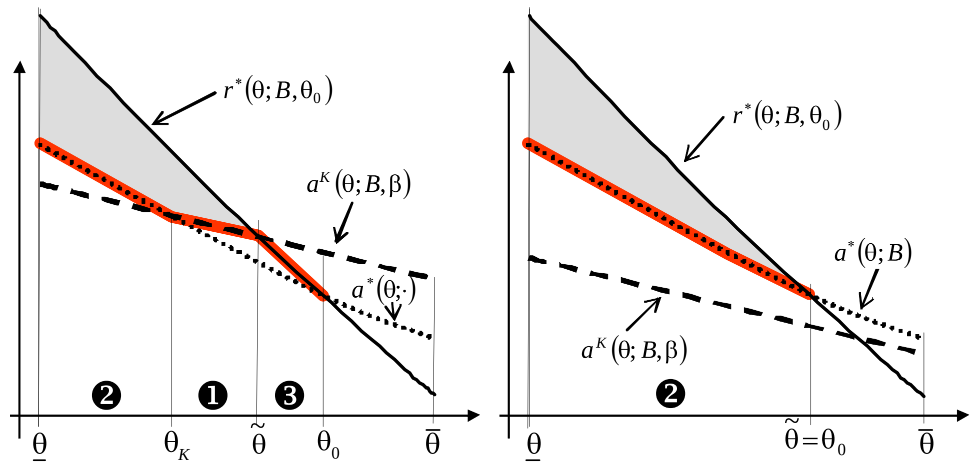

Figure 4: Applied and basic research. Low social value of applied research.

the solid thick curve is steeper that the indifference curve at the boundary of the feasible set.

5 Remarks

5.1 Low social value of applied research

I end the paper with three observations. The first sketches how the analysis changes when Assumption 2 is violated, making the social value of applied research sufficiently low, so that the curve $a^K(\cdot)$ is below $a^*(\cdot)$ at $\theta = \underline{\theta}$, as illustrated in Figure 4. If a solution exists, it will still satisfy Proposition 3 and Corollary 2: the most productive research institutions do carry out their preferred level of applied research. However, if there are institutions which are given an incentive to do more than this, as in the right hand side panel, they are the medium θ institutions.

If the relative position of the various curves is as depicted in the RHS panel of Figure 4, then the optimal policy is implemented simply by a policy of constant funding: all research institutions that agree to carry out at least $a^*(\theta_0; B)$ applied research, receive the funds necessary to pay for it, which they can then use in any way they choose. In this case the diagram of the funding schedule looks exactly the same as the initial portion of the thick solid line on

Figure 2, from $a^* (\theta_0; B)$ to $a\left(\tilde{\theta}\right)$.

As I mentioned, the social value of research might be lowered by international spillovers. It seems plausible that a small country would be less able to internalise the benefits of applied research. The discussion of this section would therefore loosely suggest that smaller countries should be more likely to adopt a constant funding scheme.

5.2 "Dual support system"

In many countries, research is funded through a dual channel funding mechanism: some funding is a lump-sum, and some is allocated on a project by project basis (for example, DBIS 2010). The implementation of the optimal mechanism derived here is in line with this principle: all institutions are offered research funding $C_0 = c(a^* (\theta_0; B), \theta_0, B)$, provided they carry out at least $a^* (\theta_0; B)$ applied research, and in addition, institutions can apply to have specific projects funded through a grant. In the model, these grants are not available to all institutions though: to qualify to apply, an institution needs to carry out at least $a^* (\theta_K; B)$ applied research with the fixed sum C_0.[15] The additional grant funding is governed by the formula

$$g(a) = C(a + a^* (\theta_K; B)) - c(a^* (\theta_0; B), \theta_0, B) \tag{23}$$

where $g(a)$ is the amount of grant awarded for agreeing to carry out a units of applied research, in addition to the qualifying level $a^* (\theta_K; B)$.

5.3 Full economic costing

As (23) makes it clear, the amount awarded as a research grant for a specific project does not cover the additional cost of the project, except possibly for very high levels of funding. Formally.

[15] While there is no formal bar to grant funding applications on institutions that carry out only a limited amount of applied research, they are hampered by the stringent requirements regarding, for example, research infrastructure and institutional support, and in practice they do receive as research grants a lower proportion of their funding than institutions carrying out more research.

Corollary 7 *Suppose $\theta_K < \tilde{\theta} = \theta_0$. There exists $\Delta > 0$ such that there exist $\theta_\Delta > \theta_K$ such that $g(\Delta) < c(a^*(\theta_K; B) + \Delta, B, \theta) - c(a^*(\theta_K; B), B, \theta)$ for every $\theta \in (\theta_K, \theta_\Delta)$.*

Graphically, consider Figure 2. According to the Corollary, the slope of the solid thick line in a neighbourhood of $a^*(\theta_K; B)$, which represents the *additional funding* received by an institution that just exceeds the qualifying level $a^*(\theta_K; B)$, is less than the slope of the frontier at the same point, which measures the *additional cost* incurred by such an institution to exceed by a small amount the qualifying level of applied research, $a^*(\theta_K; B)$.

In words, the additional funding does not cover the extra cost of grant funded applied research, which is therefore "co-funded" by the grant funding agency and the institution. This can be compared with the practice of full economic costing, adopted, among others, by the research councils in the UK (RCUK/UUK 2010): the amount of funding for a research grant is calculated to exceed the cost to carry out the research it intends to fund. The rationale for this mechanism is that the additional funds cover the institution's fixed cost, thus avoiding cross-subsidisation among an institution's activities. My results here do not lend support to this rationale. The optimal policy is more subtle: the cross-subsidisation it entails, is in line with the principle of designing incentives to delegate funding decisions to the economic agents possessing relevant private information.

Note, however, that the argument underlying Corollary 7 does not apply if curve $C(a)$ is concave, and may anyway be reversed for higher values of applied research: very expensive applied research projects, which are carried out by very efficient institutions, may require funding that exceeds their cost.

6 Concluding Remarks

Developed countries spend around one fifth of their R&D expenditure on basic research (Gersbach 2009, p 114). Should they spend more? Less? The UK government funds research via two separates channels, quality related funding and research grants from the research councils, in a proportion of roughly 2/3 and 1/3. Is this ratio "right"? Also, research grants are less evenly distributed:

the top 25 universities UK received 85% of the research grant funding, and 75% of quality related funding. Are these proportions "right", or would society be better off with a different distribution of the same aggregate amount? Government agencies typically award research grants on a cost plus principle, whereas charitable bodies favour co-funding of research activities. Which is better?

The theoretical guidance necessary to answer these questions, and more generally to establish a microeconomic foundation to any empirical study of research funding is relatively scant. In this paper, I offer a framework for the provision of this guidance. I develop a model built on the ideas that research institutions can devote their research effort to basic or applied research, that successful applied research, unlike basic research, is observable, that there are differences in research productivity among institutions, and that the government aims to distribute its funding in the socially preferred manner, which in general differs from the preferences of individual institutions.

References

Aghion, P., Dewatripont, M., Stein, J. C., 2008. Academic freedom, private-sector focus, and the process of innovation. RAND Journal of Economics 39, 617–635.

Arrow, K., 1962. The economic implications of learning by doing. Review of Economic Studies 29, 155–173.

De Fraja, G., Valbonesi, P., 2012. The design of the university system. Journal of Public Economics 96, 317–330.

Department for Business, Innovation and Skills, 2010. The allocation of science and research funding 2011/12 to 2014/15. Department for Business, Innovation and Skills, December, London UK.

du Sautoy, M., 2003. The Music of the Primes. HarperCollins, London.

Edelson, E., 1992. Physics: Quasicrystals and superconductors: Advances in condensed matter physics. In: Greenwood, A. (Ed.), Science at the Frontier. National Academy Press, Washington DC, pp. 233–254.

Evenson, R. E., Kislev, Y., 1976. A stochastic model of applied research. Journal of Political Economy 84, 765–782.

Gersbach, H., 2009. Basic research and growth policy. In: Foray, D. (Ed.), The New Economics of Technology Policy. Edward Elgar, Cheltenham, UK, pp. 113–121.

Gersbach, H., Schneider, M., Schneller, O., 2010. Optimal mix of applied and basic research, distance to frontier, and openness. CEPR Discussion Paper 7795, London.

Gersbach, H., Sorger, G., Amon, C., 2009. Hierarchical growth: Basic and applied research. ETH Economics Working Paper Series 09/118, Zurich.

Griliches, Z., 1986. Productivity, R&D, and basic research at the firm level in the 1970's. American Economic Review 76, 141–154.

Haustein, M., 2009. Effects of the theory of relativity in the GPS. Chemnitz University of Technology.

HEFCE, 2011. Decisions on assessing research impact. Higher Education Funding Council for England, March, London UK.

Jensen, R., Thursby, M., 2001. Proofs and prototypes for sale: The licensing of university inventions. American Economic Review 91, 240–259.

Laffont, J.-J., Tirole, J., 1993. A Theory of Incentives in Procurement and Regulation. MIT Press, Cambridge, Massachusetts.

Leonard, D., van Long, N., 1992. Optimal Control Theory and Static Optimization in Economics. Cambridge University Press, Cambridge, UK.

Link, A. N., 1981. Basic research and productivity increase in manufacturing. American Economic Review 71, 1111–1112.

Macho-Stadler, I., Pérez-Castrillo, D., Veugelers, R., 2007. Licensing of university inventions: The role of a technology transfer office. International Journal of Industrial Organization 25, 483–510.

Mansfield, E., 1980. Basic research and productivity increase in manufacturing. American Economic Review 70, 863–873.

Moody, R. V., 1995. Why curiosity driven research? http://www.math.mun.ca /∼edgar/moody.html.

National Science Board, 2008. Research and development: Essential foundation for U.S. competitiveness in a global economy. National Science Foundation.

Nelson, R. R., 1959. The simple economics of basic scientific research. Journal of Political Economy 67, 297–306.

OECD, 2009. OECD science, technology and industry scoreboard 2009. OECD, Paris.

Palomino, F., Sákovics, J., 2004. Inter-league competition for talent vs. competitive balance. International Journal of Industrial Organization 22, 783–797.

RCUK/UUK Task Group, 2010. Financial sustainability and efficiency in full economic costing of research in UK higher education institutions. London.

Stephan, P. E., 1996. The economics of science. Journal of Economic Literature 34, 1199–1235.

Strandburg, K. J., 2005. Curiosity-driven research and university technology transfer. The Berkeley Electronic Press.

Appendix

Proof of Proposition 1. Divide the government objective function (7) by $(1+\lambda)$, substitute (8) and the value of T to write the optimization problem as:

$$\max_{a(\theta),r(\theta),A,B} \frac{Y(A)+k(A+B)}{1+\lambda} - \int_{\underline{\theta}}^{\bar{\theta}} \left[c(a(\theta),\theta,B)+r(\theta)-a(\theta)\right] f(\theta)\, d\theta, \quad \text{(A1)}$$

s.t. (6) and (8).

Next, following Leonard and van Long (1992, p 190), write (8) and (6) as

$$\dot{b}^0(\theta) = [r(\theta)-a(\theta)] f(\theta), \qquad b^0(\underline{\theta}) = 0, \qquad b^0(\bar{\theta}) = B, \quad \text{(A2)}$$

$$\dot{a}^0(\theta) = a(\theta) f(\theta), \qquad a^0(\underline{\theta}) = 0, \qquad a^0(\bar{\theta}) = A. \quad \text{(A3)}$$

Ignoring for the moment the constraint $r(\theta)-a(\theta) \geqslant 0$, the Lagrangian for (A1) is:

$$\mathcal{L}(\cdot) = -\left[c(a(\theta),\theta,B)+r(\theta)-a(\theta)\right] f(\theta) + \left[\sigma a(\theta) + (1-\beta)(r(\theta)-a(\theta))\right] f(\theta),$$

where σ and $(1-\beta)$ are the Lagrange multipliers for constraints (A3) and (A2). I write the multiplier of (A2) as $(1-\beta)$ to lighten notation. The first order conditions give (see Leonard and van Long, 1992, Theorem 7.11.1, p 255):

$$\frac{\partial \mathcal{L}}{\partial a(\theta)} = \left[-c_a(a(\theta),\theta,B)+1+\sigma-(1-\beta)\right] f(\theta) = 0, \quad \text{(A4)}$$

$$\frac{\partial \mathcal{L}}{\partial r(\theta)} = (-1+(1-\beta)) f(\theta) = 0, \quad \text{(A5)}$$

$$\sigma = \frac{k+Y'(A)}{1+\lambda}, \quad \text{(A6)}$$

$$1-\beta = \frac{k}{1+\lambda} - \int_{\underline{\theta}}^{\bar{\theta}} c_B(a(\theta),\theta,B) f(\theta)\, d\theta. \quad \text{(A7)}$$

(A5) implies, $\beta = 0$, and the result follows. ■

Proof of Corollory 1. The problem in this case is the same as (A1), with the added constraint $a(\theta) \geqslant a^*(\theta; B)$. At the solution of problem (A1) derived in Proposition 1, this constraint is slack and so the solution found there remains a solution for the new problem. ■

Proof of Proposition 2. Notice first of all that $b(\theta)$ must be non-negative, and so (17) must hold. For policy $\{r(\theta), a(\theta)\}$ to be incentive compatible, every type θ institution must (weakly) prefer to report to be of type θ that to pretend of to be of type x, for every $x \in [\underline{\theta}, \bar{\theta}]$. This determines (14). To see how, begin to note that by choosing to report type $x \in [\underline{\theta}, \bar{\theta}]$, institution of type θ receives an amount of funds for applied research $a(x)$ and a total research funding $c(a(x), x, B) + r(x) - a(x) = c(a(x), x, B) + b(x)$. Total cost is observable, and so the institution needs to choose research levels such that its total cost equals the last expression. So if a institution of type θ has reported type x, it will choose research levels a_L and b_L (L stands for "lying") which maximise

$$
\begin{aligned}
(a_L, b_L) = \arg\max_{a,b}\, & a + b, \\
\text{s.t.: } & c(a, \theta, B) + b = c(a(x), x, B) + r(x) - a(x), \\
& a \geqslant a(x).
\end{aligned}
$$

Or

$$
\begin{aligned}
a_L = \max_a\, & a + c(a(x), x, B) + r(x) - a(x) - c(a, \theta, B), \\
\text{s.t.: } & a \geqslant a(x).
\end{aligned}
$$

Which has solution $a_L = a^*(\theta; B)$ if $a^*(\theta; B) \geqslant a(x)$, and $a_L = a(x)$ if $a^*(\theta; B) < a(x)$. That is:

$$
\begin{aligned}
a_L &= \max\{a^*(\theta; B), a(x)\}, \\
b_L &= c(a(x), x, B) + r(x) - a(x) - c(a_L, \theta, B),
\end{aligned}
$$

and yields payoff:

$$
\varphi(x, \theta) = \begin{cases} a^*(\theta; B) + c(a(x), B, x) + r(x) - a(x) - c(a^*(\theta; B), \theta, B), \\ \qquad \text{if } a^*(\theta; B) \geqslant a(x), \\ a(x) + c(a(x), x, B) + r(x) - a(x) - c(a(x), \theta, B), \\ \qquad \text{if } a^*(\theta; B) < a(x); \end{cases}
$$

consider local maxima:

$$
\frac{\partial \varphi(x, \theta)}{\partial x} = \begin{cases} (c_a(a(x), x, B) - 1)\, \dot{a}(x) + c_\theta(a(x), x, B) + \dot{r}(x), \\ \qquad \text{if } a^*(\theta; B) \geqslant a(x), \\ c_a(a(x), x, B)\, \dot{a}(x) + c_\theta(a(x), x, B) + \dot{r}(x) - c_a(a(x), \theta, B), \\ \qquad \text{if } a^*(\theta; B) < a(x). \end{cases}
$$

For incentive compatibility, $\varphi(x, \theta)$ needs to be maximised at the true type $x = \theta$. Evaluating the above at $x = \theta$, we get:

$$
\frac{\partial \varphi(x, \theta)}{\partial x} = \begin{cases} c_\theta(a(x), x, B) + \dot{r}(x) & \text{if } \quad a^*(\theta; B) \geqslant a(x), \\ c_\theta(a(x), x, B) + \dot{r}(x) & \text{if } \quad a^*(\theta; B) < a(x). \end{cases}
$$

The first line holds because the optimal a when $a^*(\theta;B) \geqslant a(x)$ is $a^*(\theta;B)$ and $c_a(a^*(\theta;B), x, B) - 1 = 0$: this establishes (14).

Now (15): following Laffont and Tirole (1993, p 121), for a policy to be incentive compatible it must be that

$$\frac{\partial^2 \varphi(\theta,x)}{\partial\theta\partial x} \geqslant 0.$$

We have

$$\frac{\partial^2 \varphi(\theta,x)}{\partial\theta\partial x} = -c_{a\theta}(a(x), \theta, B)\,\dot{a}(x) \geqslant 0,$$

given our assumption that $c_{a\theta}(a(x), \theta, B) > 0$, (15) must hold.

Finally, (16). This follows from

$$\frac{d\left(c(a(\theta), \theta, B) + r(\theta) - a(\theta)\right)}{d\theta} \leqslant 0. \tag{A8}$$

This is the constraint that total funding be decreasing in θ. If it were not the case, then an institution could simply claim to have a higher θ than it has, thus receiving a higher funding, which it could spend on unobservable basic research. Expand (A8):

$$c_a(a(\theta), \theta, B)\,\dot{a}(\theta) + c_\theta(a(\theta), \theta, B) + \dot{r}(\theta) - \dot{a}(\theta) \leqslant 0$$

which, using (14), becomes

$$[c_a(a(\theta), \theta, B) - 1]\,\dot{a}(\theta) \leqslant 0,$$

since $\dot{a}(\theta) \leqslant 0$, $c_a(a(\theta), \theta, B)$ must exceed 1, which is (16). ■

Proof of Proposition 3. Begin by noting that $a(\theta) \in [a^*(\theta;B), r(\theta)]$, and therefore a solution exists only for values of θ such that $r(\theta) \geqslant a^*(\theta;B)$, that is for $\theta \leqslant \theta_0$ at the candidate solution. This is because, by virtue of Assumption 3(i), $r(\theta) > a^*(\theta;B)$ to the left of their intersection, θ_0.

I proceed as in Proposition 1: divide the maximand of problem (18) by $(1+\lambda)$, and construct the Lagrangian.

$$\begin{aligned}\mathcal{L}(\cdot) = &-[c(a(\theta), \theta, B) + r(\theta) - a(\theta)]\,f(\theta) - \mu(\theta)\,c_\theta(a(\theta), \theta, B) + \\ &\gamma(\theta)(a(\theta) - a^*(\theta;B)) + \pi(\theta)(r(\theta) - a(\theta)) + \\ &[(1-\beta)(r(\theta) - a(\theta)) + \sigma a(\theta)]\,f(\theta),\end{aligned} \tag{A9}$$

where $\mu(\theta)$, $\gamma(\theta)$, $\pi(\theta)$, are the multipliers associated to constraints (14), (16), and (17) respectively. As before $(1-\beta) > 0$ and $\sigma > 0$ are the multipliers for (A2) and

(A3). I have ignored constraint (15): it will be seen to be satisfied at the solution found when it is ignored. The first order conditions on $r(\theta)$ and $a(\theta)$ are given by:

$$-\frac{\partial \mathcal{L}}{\partial r(\theta)} = \dot{\mu}(\theta) = \beta f(\theta) - \pi(\theta), \qquad \mu(\underline{\theta}) = 0, \qquad \mu(\bar{\theta}) \text{ free;} \tag{A10}$$

$$\frac{\partial \mathcal{L}}{\partial a(\theta)} = [-c_a(a(\theta), \theta, B) + \beta + \sigma] f(\theta) + \gamma(\theta) - \pi(\theta) - \mu(\theta) c_{\theta a}(a(\theta), \theta, B) = 0. \tag{A11}$$

(A10) has solution:

$$\mu(\theta) = \beta F(\theta) - \Pi(\theta), \tag{A12}$$

having defined $\Pi(\theta) = \int_{\underline{\theta}}^{\theta} \pi\left(\tilde{\theta}\right) d\tilde{\theta}$. The first order conditions for A and B are the same as in Proposition 1, giving $\sigma = \frac{k+Y'(A)}{1+\lambda}$. Expand the condition on $(1-\beta)$, using (A12), and the definition of $a^*(\theta; B)$, which implies $\frac{\partial a^*}{\partial B} = -\frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}$:

$$1 - \beta = \frac{k}{1+\lambda} + \int_{\underline{\theta}}^{\theta_0} \left[-c_B(\cdot) f(\theta) - (\beta F(\theta) - \Pi(\theta)) c_{\theta B}(\cdot) + \gamma(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}\right] d\theta.$$

Integration by parts gives:

$$1 - \beta = \frac{k}{1+\lambda} - c_B(a(\theta_0), \theta_0, B) F(\theta_0) + (1-\beta) \int_{\underline{\theta}}^{\theta_0} F(\theta) c_{\theta B}(\cdot) d\theta + \int_{\underline{\theta}}^{\theta_0} \left[\Pi(\theta) c_{\theta B}(\cdot) + \gamma(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}\right] d\theta,$$

and so

$$1 - \beta = \frac{\frac{k}{1+\lambda} - c_B(a(\theta_0), \theta_0, B) F(\theta_0)}{1 - \int_{\underline{\theta}}^{\theta_0} F(\theta) c_{\theta B}(\cdot) d\theta} + \frac{\int_{\underline{\theta}}^{\theta_0} \left[\Pi(\theta) c_{\theta B}(\cdot) + \gamma(\theta) \frac{c_{aB}(\cdot)}{c_{aa}(\cdot)}\right] d\theta}{1 - \int_{\underline{\theta}}^{\theta_0} F(\theta) c_{\theta B}(\cdot) d\theta}. \tag{A13}$$

From (A11) we obtain the optimality condition for $a(\theta)$.

$$c_a(a(\theta), \theta, B) = \frac{Y'(A) + k}{1+\lambda} + \beta + \frac{\gamma(\theta) - \pi(\theta)}{f(\theta)} - \frac{\beta F(\theta) - \Pi(\theta)}{f(\theta)} c_{\theta a}(a(\theta), \theta, B). \tag{A14}$$

Next I show that $\beta \geqslant 0$. To see this, notice that $(1-\beta)$ measures the benefit of relaxing the constraint $b(\theta) \geqslant 0$, which has a cost of 1, measured in the social value of monetary units. Notice that the funding agency can always increase $b(\theta)$ if it wants, because it can simply increase the funding to all research institutions, and, since at the optimum they all do at least $a^*(\theta; B)$, they all prefer to spend the additional funding on basic research. Therefore the benefit of increasing $b(\theta)$ cannot exceed the cost at the optimum: $(1-\beta) \leqslant 1$.

Now define the function $a^K_\Pi(\theta; B, \beta)$ as the solution in a of

$$c_a(a, \theta, B) = \frac{Y'(A) + k}{1 + \lambda} + \beta - \frac{\beta F(\theta) - \Pi}{f(\theta)} c_{\theta a}(a, \theta, B). \tag{A15}$$

If $\Pi = 0$, then $a^K_\Pi(\theta; B, \beta) = a^K(\theta; B, \beta)$ and if $\Pi > 0$, then $a^K_\Pi(\theta; B, \beta) > a^K(\theta; B, \beta)$, since $c_{\theta a}(\cdot) > 0$.

Next notice that depending on the combination of complementary slackness for constraints (16) and (17), a value of $a(\theta)$ belongs to one of four possible regions:

1. $a(\theta) - a^*(\theta; B) > 0$ and $r(\theta) - a(\theta) > 0$. Therefore, $\gamma(\theta) = \pi(\theta) = 0$, which means $r(\theta) > a(\theta) > a^*(\theta; B)$, and in this region, $r(\theta) = r^*(\theta; B, \theta_0)$, $a(\theta) = a^K_{\Pi(\theta)}(\theta; B, \beta)$.

2. $\gamma(\theta) > 0$ and $r(\theta) - a(\theta) > 0$. Here, $a(\theta) - a^*(\theta; B) = 0$ and $\pi(\theta) = 0$, and so $r(\theta) = r^*(\theta; B, \theta_0)$, $a(\theta) = a^*(\theta; B)$.

3. $a(\theta) - a^*(\theta; B) > 0$ and $\pi(\theta) > 0$. In this region $\gamma(\theta) = 0$ and $r(\theta) = a(\theta) = r^*(\theta; B, \theta_0)$.

4. $\gamma(\theta) > 0$ and $\pi(\theta) > 0$. Here, $r(\theta) = r^*(\theta; B, \theta_0) = a^*(\theta; B) = a(\theta)$, and therefore this region is just the single intersection point between $a^*(\theta; B)$ and $r^*(\theta; B, \theta_0)$.

As a preliminary step, I show that

$$\begin{aligned} &\text{if } \theta \in \left[\underline{\theta}, \tilde{\theta}\right) \text{ then } a(\theta) > 0 \text{ and } b(\theta) > 0; \\ &\text{if } \theta \in \left[\tilde{\theta}, \theta_0\right] \text{ then } a(\theta) > 0 \text{ and } b(\theta) = 0. \end{aligned}$$

Proposition 3 requires that $\underline{\theta}$ belongs to regions 1 or 2, that is that $a(\underline{\theta}) \in [a^*(\underline{\theta}, B), r^*(\underline{\theta}; B, \theta_0))$. Suppose by contradiction that $a(\underline{\theta}) = r^*(\underline{\theta}; B, \theta_0)$. Then $b(\underline{\theta}) = 0$ in $\left[\underline{\theta}, \tilde{\theta}\right]$ for some $\tilde{\theta} > \underline{\theta}$. Notice next that it cannot be $\tilde{\theta} = \theta_0$, otherwise $b(\theta) = 0$ in $[\underline{\theta}, \bar{\theta}]$ and so $B = 0$, against the Inada condition. That is, there is $\tilde{\theta} > \theta_0$ such that $a(\theta) = a^K_{\Pi(\theta)}(\theta; B, \beta) < r^*(\theta; B, \theta_0)$ in a right neighbourhood of $\tilde{\theta}$, with of course $a\left(\tilde{\theta}\right) = r^*\left(\tilde{\theta}; B, \theta_0\right) = a^K_{\Pi(\theta)}\left(\tilde{\theta}; B, \beta\right)$. Now we show that at any intersection between $r^*(\theta; B, \theta_0)$ and $a^K_{\Pi(\theta)}(\theta; B, \beta)$ the latter is less steep than $r^*(\theta; B, \theta_0)$, and thus we obtain a contradiction: if $a^K_{\Pi(\theta)}(\theta; B, \beta)$ is less steep than $r^*(\theta; B, \theta_0)$ then it must be above it in a right neighbourhood of $\tilde{\theta}$.

Lemma A1 *$a^K_\Pi(\theta; B, \beta) > r^*(\theta; B, \theta_0)$ for $\theta > \tilde{\theta}$.*

Proof. To see this, compare $a_{\Pi}^{K}(\theta, B, g)$ and $r^{*}(\theta; B, \theta_0)$ at their intersection. Since we are assuming that $a(\theta) > r^{*}(\theta; B, \theta_0)$, we have $\pi(\theta) = \frac{d\Pi}{d\theta} = 0$. Moreover, since $a(\theta)$ is above $a^{*}(\theta; B)$ in $\left[\underline{\theta}, \tilde{\theta}\right]$, it must be $\beta F(\theta) - \Pi(\theta) > 0$ in $\left[\underline{\theta}, \tilde{\theta}\right]$. Next totally differentiate (A15):

$$\begin{aligned}&\left[c_{aa}(\cdot) + \frac{gF(\theta) - \Pi}{f(\theta)} c_{\theta aa}(\cdot)\right] da + \\ &\left[c_{a\theta}(\cdot) + \frac{gF(\theta) - \Pi}{f(\theta)} c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)\, g \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)\right] d\theta = 0.\end{aligned}$$

Hence:

$$\frac{\partial a_K(\theta; B, g)}{\partial \theta} = -\frac{c_{a\theta}(\cdot) + \frac{gF(\theta)-\Pi}{f(\theta)} c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)\, g \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{c_{aa}(\cdot) + \frac{gF(\theta)-\Pi}{(1+\lambda) f(\theta)} c_{\theta aa}(\cdot)}.$$

I need to verify that the following holds:

$$-\frac{c_{a\theta}(\cdot) + \frac{gF(\theta)-\Pi}{f(\theta)} c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot)\, g \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{c_{aa}(\cdot) + \frac{gF(\theta)-\Pi}{f(\theta)} c_{\theta aa}(\cdot)} > -c_{\theta}(\cdot).$$

By Assumption 3, $c_{\theta aa}(\cdot) > 0$, and so I can multiply through and rearrange:

$$\frac{\frac{gF(\theta)-\Pi}{f(\theta)}}{c_{aa}(\cdot)} \left(c_{\theta\theta a}(\cdot) + c_{a\theta}(\cdot) \frac{\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)}{\frac{F(\theta)}{f(\theta)}} - c_{\theta aa}(\cdot)\, c_{\theta}(\cdot) \right) < c_{\theta}(\cdot) - \frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)}.$$

Again, by Assumption 3, the RHS is positive and the LHS is negative. Therefore, at their intersection, $\frac{\partial a_K(\cdot)}{\partial \theta} > \frac{\partial r^{*}(\cdot)}{\partial \theta}$, that is $r^{*}(\cdot)$ is steeper, and so it is below $a^{K}(\cdot)$ in a right neighbourhood of their intersection. The contradiction establishes the Lemma. ■

The Proposition now follows immediately. ■

Proof of Corollary 2. Proposition 3 shows that $a(\theta)$ is one of $a^{*}(\theta; B)$, $a^{K}(\theta; B, \beta)$ or $r^{*}(\theta; B, \theta_0)$. Moreover, since it must lie between $a^{*}(\theta; B)$ and $r^{*}(\theta; B, \theta_0)$, it can only equal $a^{K}(\theta; B, \beta)$ – intersections excepted – between them. The second line follows from the first. ■

Proof of Corollary 3. Omitted. ■

Proof of Corollary 4. Differentiate (21) with respect to a, using (14):

$$C'(a) = c_a\left(a, \theta^{K}(a; B, \beta), B\right) - 1. \tag{A16}$$

The above is positive because $a^K(\theta; B, \beta)$ exceeds $a^*(\theta; B)$. C is therefore increasing. For the second part of the statement, expand $C''(a)$:

$$C''(a) = c_{aa}(\cdot) + c_{a\theta}(\cdot)\frac{\partial \theta^K(a; B, \beta)}{\partial a}.$$

This is positive if $-\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} = \frac{\partial a^*(\theta;B)}{\partial \theta} > \frac{\partial a^K(\theta;B,\beta)}{\partial \theta}$. ∎

Proof of Corollary 5. The derivative of (22) is:

$$C'(a) = c_a(\cdot) + c_\theta(\cdot)\frac{\partial \theta^*(a; B)}{\partial a} + \frac{\partial r^*(\theta^*(a; B); B, \theta_0)}{\partial \theta}\frac{\partial \theta^*(a; B)}{\partial a} - 1 = 0,$$

as $c_a(\cdot) = 1$ along $a^*(\theta; B)$. ∎

Proof of Corollary 6. Let $\theta^r(a; B, \theta_0)$ be the inverse function of $r^*(\theta; B, \theta_0)$, and total funding is given by (recall that $b(\theta) = 0$ in this region):

$$C(a) = c(a, \theta^r(a, B, \theta_0), B). \tag{A17}$$

Differentiation with respect to a yields:

$$C'(a) = c_a(\cdot) + \frac{c_\theta(\cdot)}{\frac{\partial r^*(\cdot)}{\partial \theta}} = c_a(\cdot) - 1.$$

Since $r^*(\theta; B, \theta_0) > a^*(\theta; B)$ except at θ_0, the above is positve in $\left(\tilde{\theta}, \theta_0\right)$. To establish convexity, take $C''(a)$:

$$C''(a) = c_{aa}(\cdot) + c_{a\theta}(\cdot)\frac{\partial \theta^r(a; B, \theta_0)}{\partial a},$$

which is positive as $-\frac{c_{a\theta}(\cdot)}{c_{aa}(\cdot)} > \frac{\partial r^*(\theta;B,\theta_0)}{\partial \theta} = -c_\theta(\cdot)$.

For the second part of the statement, note that, in region 3 (that is to the left of their intersection), the slope of $C(a)$ is $c_a(a, \theta^r(a; B, \theta_0), B) - 1$. In region 1, namely to the right of their intersection, the slope is $c_a\left(a, \theta^K(a; B, \beta), B\right) - 1$. Consider a right neighbourhood of their intersection: the difference in slope is

$$\begin{aligned} & c_a(a, \theta^r(a; B, \theta_0), B) - c_a\left(a, \theta^K(a; B, \beta), B\right) \\ & \qquad = c_{a\theta}(a, \theta_3, B)\left(\theta^r(a; B, \theta_0) - \theta^K(a; B, \beta)\right). \end{aligned} \tag{A18}$$

This is positive, since $\theta^r(a; B, \theta_0) - \theta^K(a; B, \beta) > 0$, establishing the statement. ∎

Proof of Corollary 7. Omitted. ∎