Detecting Bid-rigging in Procurement Auctions: Three-step Approach to Reveal Conspiracy[1]

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Collusion is a hot topic. Even in private sector owner should design the procedure to break possible collusive behavior of the agents. The problem is more acute in state sector where purchasers of goods lack of incentives to maintain healthy competition. Highway construction is traditionally attractive for collusion because of high potential spoils. The quality of road works is very hard to verify, so firms usually have lot of abilities to economize, for instance, by the usage of cheaper raw materials. It increases potential gains which can be divided either inside the cartel (among its members) or between contractors and government purchaser. The main idea of our paper is to examine seven-step approach of detecting collusion suggested in the article of Padhi and Mohaparta on Russian dataset.

1. Introduction

There is a group of authors who suggested various methods of collusion detection. Unfortunately, practically all of these methods required rich data sets or were based on a priori knowledge about the presence of conspiracy, and only a couple of works offered an approach which allows to reveal collusive schemes 'from scratch', merely having an information on the winning bids in the auctions conducted. One of these works, which generalizes previous research experience in this area, is the article of Padhi and Mohaparta (2011).

The main idea of our paper is to examine seven-step approach of detecting collusion suggested in the article of Padhi and Mohaparta. We do not precisely follow authors' logic in this analysis: we neglect some steps like testing hypothesis of equality variances and means because we find them statistically incorrect. So in fact we use modified Padhi and Mohaparta's approach. We also present different way of drawing conclusions after an explanation of the results and show why the way of drawing conclusions suggested by the above-mentioned authors can be inaccurate for some sets of data. We show that assisted with our three-step method of collusion detection,

researcher can reveal not only 'obvious' procedures, where conspiracy can exist, but also some unconventional cases, where, for instance, participants of the race used to submit phony bids or where non-cartel firms competed with the cartel.

So the paper is organized as follows: in Section 2 we present literature review connected with various methods of collusion detection suggested in the previous researches. We present our data set in Section 3. Then, Section 4 contains all information about suggested three-step statistical analysis and offers conclusions. Section 5 sets a couple of open questions and ends the paper with some concluding remarks.

2. Literature review

Why detection of collusion is a hard task?

One who tries to detect collusive schemes in the particular market may face with a couple of obnoxious problems. First, it is worth mentioning that conspiracy does not always imply physical coordination between its participants. In conditions of information asymmetry, when firms do not exactly know the valuations of other players, submitting a bid equal to the reserve price can be the optimal strategy[1]. In such case we would observe n firms submitted n equal bids, and if this strategy brings them positive profit (in situation, for example, when in case of equal bids an auctioneer awards the contract randomly), we will observe cartel prices in the market for a long (McAfee and McMillan, 1992). There is a large amount of the empirical researches illustrating this phenomenon, which is also known as tacit collusion. Some of them describe the paradoxical equality of bids in sealed-bid auctions in USA, Canada and some European countries (Mund, 1960; Cook, 1963; Comanor and Schankerman, 1976).

The second key problem which impedes cartel detecting is so called phony bidding. In order to reduce risks of being prosecuted, members of conspiracy can submit fake bids, which are extremely close to the maximum bid (for an ascending auction) of pre-arranged winner. To coordinate, conspiracy members frequently participate in pre-auction meetings in order to define the winners of all nearest procedures. These meetings might be organized, for example, as first-price auctions so as to define the lowest cost firm. The highest conspirator bid is likely to be submitted by the firm with the lowest costs, which guarantees the highest potential rent which is to be divided among cartel members (Porter and Zona, 1993). Such strategy not only maximizes cartel's profit, but also makes defection from conspiracy strategy less attractive. Thus, cartel members both get the contract at the maximum price and create an illusion that they perform as competitive market does. Moreover, phony bidding not only draws away anti-trust agency's attention, but also sends a signal to the auctioneer that costs are high and the initial price is therefore under-valued (Feinstein et al.,1985).

Approaches and methods

Many authors suggested various methods of collusion detection. We point out two groups of researches here: the first is concentrated on the bids distribution and thus offers methods of structural analysis of costs and submitted bids, while the second focuses mainly on winning bids and tries to reveal deviations from the theory of competitive pricing in a set of auctions.

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[1] The analogy is widely known "Prisoners Dilemma" where Bertrand Game paradox can also be resolved in the repeatedly interacting game. In our case Pareto optimum is the situation when all bid at the level of initial price, and according to the Folk Theorem it can be equilibrium of the game under consideration.

Structural analysis of bidding patterns implies assumptions about the distributions of values. The idea is that if researcher has information about all submitted bids, he is able to compare real distribution with one drawn from the theoretical model. For example, Baldwin et al. (1997) suggest an econometric approach to collusion detection among a set of English auctions bidders. To estimate their structural model authors use data on submitted bids in forest timber purchase auctions. Research is based on the second-price sealed bid auction theory. If the environment is competitive, the winning bid in the auction with private values is a second-lowest statistic of value distribution. But if collusion appears, winning bid might fluctuate. Thus in the case when bidders with the first and the second lowest values become cartel members – winning bid can rise sharply to the lowest non-cartel firm's valuation. When they remain competitive – winning bid won't change.

The theory of competitive bidding can offer even stronger assumptions about bids distribution. For instance, submitted bids in the competitive auctions should not be correlated and should be fully defined by costs. These two assumptions are also known as conditional independence and exchangeability and were suggested in Bajari and Summers (2002), and Bajari and Ye (2003). These two conditions were treated by the authors as 'necessary and sufficient'. The first condition implies that bids can depend on each other only if participants who submitted these bids are members of collusion. To test this condition Bajari and Ye carry out correlation tests and analyze matrix of simultaneous bids. For their data set on the seal coat industry auctions they found that only a couple of pairs of firms appeared not to satisfy condition of conditional independence. These firms, for that matter, were previously sanctioned for collusion in this market. The second condition – exchangeability – stands for the dependency of the bids on production costs. The assumption is that costs alone should determine how high the firm bids. Authors constructed regression model and carried out F-test in order to calculate residuals' correlation. Again all except three previously sanctioned firms appeared to satisfy exchangeability condition.

Thus the assumptions about values and therefore bids distribution might be useful for collusion detection. However, to apply such methods one should have full information about all bids of all auction bidders. This data might be unavailable. For example, in Russian regional sites of government procurements only the first or, in some cases, the first and the second lowest bids are published. Thus we are limited in using such methods of conspiracy detection.

The second group of articles focuses on the determinants of winning bids. The idea is intuitively clear: we define factors which can influence winning bid and test control group of competitive procedures in order to reveal a mechanism of normal winning bid formation. If we then check a subset of collusive auctions, we will certainly observe serious deviations from the revealed mechanism.

For instance, Hendricks and Porter (1988) analyze drainage auctions. In the offshore gas and oil drainage auctions, number of firms owing the mineral rights and thus having strong competitive advantages is limited. Moreover, the identities of these firms are widely known, that facilitates collusion even more. Thus incumbents in this market are not afraid of entry and do not have difficulties with coordination, so they are likely to create a cartel. Carrying out regression analysis, Hendricks and Porter reveal that the winning bids in drainage auctions with incumbents do not depend on the number of bidders, and the average value of bid is likely to be a decreasing

function of participants' number. This fact is consistent with submitting only one serious bid and creating appearance of competition by phony bidding.

Similar logic is used in Porter and Zona's (1999) case of Ohio school milk markets. Authors provide evidence, that the bidding behavior of some firms is likely to be collusive, as it contradicts theoretical strategy of competitive bidding. Their econometric analysis showed that distant bids were comparatively low, while in the local auctions firms submitted much higher bids. That is the opposite of what we should see in the competitive environment. To prove an obvious statement that distant bids should be higher because of transportation costs, Porter and Zona tested a couple of control groups which are likely to be competitive and found that bid is really an increasing function of the distance. Thus they illustrated, that bidding patterns of those firms which deviated from competitive bidding strategy are consistent with territorial allocation of school milk supply contracts.

An assumption that collusion members bid less aggressively than non-cartel firms was also proved in Pesendorfer (2000). To find the proof, author used his knowledge about the presence of two cartels in Florida and Texas school milk markets. Pesendorfer tested for the asymmetries between bidders in two groups – cartel and non-cartel firms. He showed that bidding rules of the firms in these two groups are significantly different. For instance, raw materials price has on average a significantly lower influence on collusion members' bids that on the bids of non-cartel firms. Similar approach was also used in Porter and Zona (1993), where the test was applied to the data on Long Island highway construction and maintenance contracts. Porter and Zona built an OLS regression model where winning price-to-reserve ratio was a dependent variable and explanatory variables were distance, experience of the firm, utilized capacity, etc. The results show that statistically different bidding behavior can be observed among cartel and non-cartel firms. Moreover, residuals to estimated functions of bid are significantly higher correlated with each other for the cartel participants than for the competitive non-cartel bidders.

These regression approaches may seem the most reliable, however it requires data not only on winning bids, but also on such parameters as distance, firms' utilized capacity and experience, some contract characteristics and so on, which might be hard to gather. Furthermore, in all above-mentioned articles authors had a priori knowledge about cartel presence. Thus if we don't know that there is a cartel or cartels in the market, it is not clear how can we detect conspiracy presence and what groups of auctions should be tested for the asymmetries.

Clustering can be a useful method of bid-rigging detection, especially if we don't have a priori knowledge about cartel presence. This method allows dividing a set of auctions into different parts. The tool is convenient mostly because it divides auctions into clusters, which are significantly different from each other. So if two patterns of bidding – collusive and competitive – are presented in the database, we have a high chance of getting two clusters – one cluster with auctions, where struggling between firms reduced price (so where price ratio mean value is low), and the other one high-value price ratio cluster, where conspiracy presence is likely to be.

This idea was used in the paper Padhi and Mohaparta (2011), where the authors suggested original seven-step analysis, based on the results of previous researches in this area. Suggested approach theoretically allows us to divide a data set into potential groups of cartel and non-cartel firms, which is extremely useful for the further analysis. All we need to apply this method is information about price-to-reserve ratios in a set of auctions.

Padhi and Mohaparta suggested using the following scheme of collusion detection:

1. *Cluster analysis.* Two-step cluster analysis was used, which resulted in clear division of data on price ratios into two clusters. The first cluster consisted of high-price-ratio contracts, while the second included auctions with comparatively high decrease in price, which are more likely to be competitive procedures.
2. *Non-parametric tests.* Padhi and Mohaparta tested hypotheses of difference of means and deviations and found that two clusters are statistically different. It also consistent with hypothesis, that in the market where collusion exists, we should theoretically observe two significantly different patterns of bidding. In addition they also use box-slippage test to prove a difference in medians.
3. Normality and skewness. On this step authors carried out Kolmogorov-Smirnov normality test and calculated skewness for each of two clusters. The low-ratio cluster (as it is expected in theory) turned out to be a normal[1] symmetric distribution of price ratios, when high-ratio group of auctions appeared to be negatively skewed and far from normal. This result proves hypothesis about being low-ratio cluster a competitive one and the other – cluster where conspiracy is likely to be. This is because collusion, if it appears, pushes prices up and creates a negatively skewed distribution of winning-bid-price ratios.
4. Autocorrelation test was also used on the last step of authors' analysis in order to look for cyclic pattern among the winners to detect collusive behavior. Authors concluded that winning bidders really followed a cyclic winning pattern with a period of three procedures. However autocorrelation test is not a focus of our paper, so we now go on with an explanation of our modified (three-step) statistical analysis.

Thus, having merely a data set on winning price ratios, authors divided auctions into two groups, one of which is consistent with collusive bidding patterns. Of course many limitations of this approach can be discussed. However, in general, suggested seven-step method can be useful for the further regression analysis[2].

3. Data set

To apply above-described seven-step method of collusion detection we use the data set on the Highway building and maintenance contracts in Novosibirsk region. The sample consists of 210 contracts awarded in 2010 by Territorial Highway Administration (THA) – the last is responsible for all state purchases in highway industry in this region. The following information about these contracts was gathered: date of the procedure, the type of construction work, initial price, the lowest bid, winner of the auction and all participants (applied; confirmed; came to the tender), deadline and local district of the road works.

Only 10 procedures were sealed-bid first price auctions, the others were open first price ones. The average number of competitors coming to the auction is 2.14, however the mode equals 1, as 120 of 210 contracts were awarded to the only participant who came to the auction. The contract

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[1] Actually, competitive or low-ratio cluster was positively skewed, which means that there was a shift towards high decrease in those auctions. However skewness was not so big, so the situation which authors observed also fits well to a theoretical framework.
[2] After an explanation of their seven-step approach of collusion detection, Padhi and Mohaparta tested their clusters by creating regression model of winning price-to-reserve ratio function. They showed that rules of winning bid formation are significantly different within two clusters.

price ratio[1] varies from 0,23 to 1,00, having the average at the level of 0,91. It is also worth mentioning that 135 of 210 contracts were awarded at the maximum price (with price ratio equals 1,00).

The total number of unique participants in the auctions in 2010 was 56. Most of them are not specialized in the narrow category of construction works and are capable to do most of the works tendered: from document preparation to the bridge and highway and construction. In Novosibirsk region firms bid for the contracts on works, which can be located in one of the 30 local districts. General statistics of the data can be seen in Table 1.

	Min	Max	Mean	Mode	Dev
Reserve price, rub	94900	262527520	25391608	20000000	6285,674
Lenght of contract, days	11	1306	364,2	942	20,013
Ratio	0,23	1	0,908	1	0,419
Number of applications	1	15	3,05	1	1,55
Number of participants	1	13	2,14	1	1,43

Table 1

4. Statistical analysis

We carry out a three-step analysis, which steps' order is similar to the order in Padhi and Mohaparta seven-step approach. However there are some differences from these authors' approach which will be discussed at the end of this section.

First step. We start with two-cluster analysis of the price ratio in order to distinguish two or more groups of auctions. That is needed to test whether there is a group of auctions with comparatively low decrease in price (with ratio equals about 1,00), which is likely to be a collusion, or not.
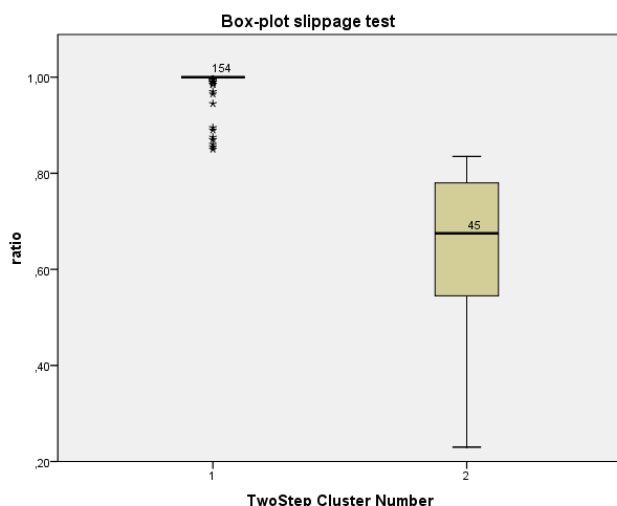

Figure 1

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[1] It is counted as the relation of contract price to initial auction price.

The result is shown in Fig.1 – we can see two clearly divided clusters of auctions. The first cluster, which is on the left, include 154 auctions (78% of all) with high price ratios, varying from 0,85 to 1,00. Thus, the winning bid prices in these auctions are concentrated near the initial prices. Moreover, deviation is practically zero as 149 of 154 auctions ended without any price decrease. As for the second cluster, in 45 auctions (22%) ratio shows value between 0,23 and 0,84. Price ratio varies much here, which is more likely to be a normal distribution (normality test will be conducted on the next step). Summary statistics of two clusters is shown in Table 2.

	Range	mean	SD
Cluster 1 (154 of 199)	0,85-1,00	0,9924	0,02758
Cluster 2 (45 of 199)	0,20-0,80	0,6337	0,16663
Combined (199)		0,9113	0,17142

Table 2

So we have two groups of auctions, which are significantly different from each other (Formally, to claim this we should check whether they have their means and deviations equal by testing appropriate hypotheses. We will get back to this issue later). Furthermore, we suggest that the first cluster includes collusive cases, thus price ratios should be far from a normal distribution in this group. On the contrary, the first cluster demonstrates competitive behavior pattern and subjects to a normal symmetric distribution.

	Kolmogorov-Smirnov		
	Statistic	df	Sig.
Cluster 1	0,433	45	0,000
Cluster 2	0,116	45	0,155

Table 3

Second step. To test whether the distribution in each cluster is close to a normal one, we carry out K-S test (Kolmogorov-Smirnov test of normality). We applied test to each of the two clusters. The results are shown in Table 3. We see that Cluster 1 price ratios are not normally distributed, while on the opposite side Cluster 2 perfectly fits normal law of distribution. This fact bolsters our supposition about the presence of collusive schemes in the first cluster – in those auctions, where collusion exists, prices are exaggerated and that is the reason for the serious deviations from normally-distributed form.

These results automatically prohibit us testing hypotheses of equality of means and deviations, which were incorrectly tested by authors. T and F-statistics can only be used when variables are distributed normally in both clusters, so it is incorrect to use them with only one normally distributed cluster[1]. However statistical difference between clusters is trivial to some extent, because cluster analysis itself implies division of the data in significantly different groups.

Third step. Next step is targeted to the same as the previous one. We study skewness of the distribution in two clusters in order to detect divergences from the theoretical models. In theory, price ratios in highly-competitive auctions should follow normal symmetric distribution (or positively skewed, that can seem strange but is not bad at all). So we calculated skewness for each of two clusters. The results are presented in Table 4.

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[1] Asymptotic approach also can't be used here because of K-S test results; despite we have large enough data set.

|	Skewness	
	Statistic	Std. Error
Cluster 1	-4,209	0,195
Cluster 2	-0,711	0,354

Table 4

We can observe that distributions in both clusters are negatively skewed – it means that prices are more likely to be higher than mean. And here we get diametrically opposed result from the one described in the above-mentioned research. There is nothing odd, of course, that high-value price ratio cluster bends forward to the higher prices, but the second (low-value ratio) cluster was expected to be symmetric or even positively skewed, which turned out to be incorrect. We will return to this strangeness later and show how this deviation can be interpreted.

Explanations. We have carried out a three-step analysis using some of the ideas proposed by Padhi and Mohaparta (2011). We split up our data in two groups – high- and low-price-ratio clusters. It gave us the reason to think over the presence of collusion in the high-price-ratio cluster. Then we demonstrated that the second (low-ratio) cluster follows a normal distribution perfectly, while the first cluster is negatively skewed and far from normal. So taking all these results into account, we now try to define which auctions are the most suspicious, and where collusion could be theoretically searched by the researcher.

Padhi and Mohaparta propose to search collusion in high-value price ratio cluster, which is in our case Cluster 1. They argue that, as second cluster is normally distributed, it includes competitive auctions and shouldn't be checked for the presence of dark schemes. In our case we cannot throw off this cluster as "competitive" because it shows some signs of collusion even if these signs are implicit.

Instead of this straightforward judgment, we suggest to mark out three groups of auctions, which are shown in Fig.2. So the groups are:

1) Auctions with price ratio equals 1,00 (without any decrease);
2) Single auctions with low decrease in price which belong to cluster 1;
3) Second cluster auctions with comparatively low decrease in price (ratio higher than mean).
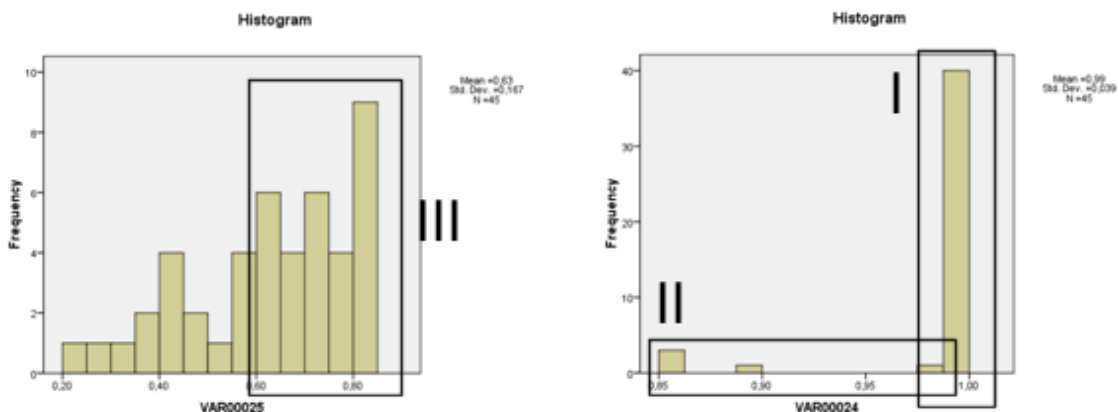
1) There is nothing new about the first (I) group of auctions that has not been previously said in the literature. Collusion implies informal 'distribution' of contracts among cartel members,

who get them one after another, in the extreme case, at the initial price. Thus the decrease in price in such cases equals zero (ratio therefore equals 1,00). So it is reasonable enough to search collusion here, but the issue is that the absence of price decrease can reflect also a low level of competition in the market but not surely collusion presence. If firms in the market are highly-specialized, we can divide it into smaller markets, and in each of them competition level will be extremely low. Thus a kind of tacit collusion is likely to appear, as equilibrium strategies of the firms will be not to come near the rivals local markets, but to get all the contracts in the own local district or of the narrow firm's specialization at high price. Thus, we will observe monopolistic prices, and that case is very hard to differ visually from physical collusion. What is more, in our data sample 149 of 199 auctions ended without decrease, so it can be a hard task to find collusion schemes in such a huge group.

2) The second suspicious group (II) includes auctions from Cluster 1 with non-unit-ratio. That is the rest of the first cluster if we throw off the first described group from it. What is strange is that these auctions seem like they belong neither to the first nor to the second cluster. Broadly speaking, it is naturally enough that data was divided into two different parts during two-step cluster analysis, because one participant in the auction does not cause any decrease, but if there are at least two participants, we should theoretically observe an abrupt shift from unit-ratio result to the decrease of the initial price by 15-20%.[1] But we have a couple of intermediate cases (ratio equals 0,9;0,95;0,995) which are small in terms of frequency and are not likely to be competitive ones from the second normally-distributed cluster[2].

The last is very important observation. High risk of being prosecuted by anti-trust agency can make all the cartel members bid in a way which strengthens the illusion of healthy competition. However decrease in price in auction with phony bids will be surely less than competitive – speaking about oral auctions, frequent strategy in Russian practice is to lower the price by the setting unit (that is 0,5% of the initial price in Russia). And that is what we can observe in the second group – there were price decreases, but these decreases are low enough to be competitive. The last is consistent with only one serious bid supported by the higher phantom bids. Of course, we can't be sure that these auctions were collusive; however we suggest analyzing this small group first of all after data gathering and carrying out cluster analysis.

3) The third group (III) is a key point in our approach partially because it was not an area of searching for the cartels in the previous researches (particularly in Padhi and Mohaparta's article). What had attracted our attention is statistically significant negative skewness of the second cluster distribution. Normal distribution becomes asymmetric only when observations correlate, and the last may be the result of the collusion presence in the market, as cartel members try to bring down prices as little as possible and thus they make distribution of price ratios skewed towards the value of 1,00. However auctions of the third group are different from those considered above – price decrease is significant here and price ratio varies nearly 0,6-0,8.

An explanation can be the following: cartel members participate in these auctions, but their bidding freedoms are limited by the presence of at least one non-cartel firm. When cartel members faces such competition, it can propose one firm to compete with outsiders and let other firms be passive during bidding. Thus winning bid is likely to be the lowest valuation

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[1] If the initial price is correctly calculated by a public body and exceeds costs 10-15% at average.

[2] One may object to this claim, that there is nothing supernatural that two bidder auction ended merely with a 5-10% decrease. However number of participants in this group was not always two, but varied from two to eight.

of non-cartel firm (Baldwin et al., 1997).[1] Obviously such auctions, where not all-inclusive cartels are presented, can potentially break the symmetry of competitive cluster's normal distribution. Therefore we suggest that a negative skewness of competitive cluster can also be a sign of cartel's presence.

To summarize, we suggested division of the data set in three different groups, while Padhi and Mohaparta propose to analyze the first group only. But, as we said, the first group is huge in comparison with the entire data set. We also know that tacit collusion frequently exists in the unit-price ratio (1,00) procedures, which is very hard to distinguish from explicit conspiracy. Thus starting analysis from this all-embracing group may be not a good idea if researcher has other suspicious groups like those, which we suggested to consider in our case (Groups I and II).

5. Discussion

In our research we supposed like other authors working in this area, that bidding patterns in competitive and collusive sets of auctions differs significantly. The idea is that price-ratios in competitive procedures should theoretically fit symmetric normal distribution, while collusive ones should not. So this theoretical statement is consistent with the results of two-step cluster analysis: set of auctions was clearly divided in two parts, which are different in terms of ratio distribution. However our competitive cluster has negative skewness, which has not been mentioned in the previous researches in this area. This fact suggested us to take aim not only at the high-price-ratio cluster, but at three different groups of 'suspicious' auctions.

However our method has some limitations. First and the most obvious is that cluster analysis (if we apply it to another data) can theoretically bring us an "extreme" result of two clusters – competitive, which is normally distributed with zero skewness, and collusive, which consists of unit-ratio auctions. In this case we will have only the first group (I) for the analysis, where distinguishing tacit and physical collusion can be an arduous task. Second problem occurs because of our implicit perquisite about the correct calculating of the reserve price by the auctioneer. Renouncing this precondition, we get additional factor which has a strong influence on the price ratio. Thus, for example, negative skewness in a competitive cluster can be the result of undervaluing of the initial price, and the same with auctions from the second group (II) with ratios equal 0,95, 0,99 etc.

Thirdly and finally, even if we have checked all suspicious auctions from the groups (I-III) and have not found any traces of collusion, it is too early to give up searching. In some cases, phony bidding can be organized by cartel members in the way that imitates normal distribution of price decreases (ratios), but prices are higher in this case than serious bids. We can't detect such cartel using our three-step approach and we even can't detect it by calculating correlation of submitted bids with costs (firms might inflate all submitted bids by the same percentage). However some other methods may shed light on this situation. For instance, it can be auto-correlation analysis that gives a way of detecting cyclicity and rotational winning patterns.

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[1] Suggest that none of non-cartel members can overbid the most efficient collusion participant.

References

Bajari, P., Summers, G., 2002. Detection of collusion in procurement auctions. Antitrust Law Journal 70 (1), 143-170.

Bajari, P., Ye, L., 2003. Deciding between competition and collusion. The Review of Economics and Statistics 85 (4), 971-989.

Baldwin, L.H., Marshall, R.C., Richard, J.F., 1997. Bidder collusion at forest service timber sales. Journal of Political Economy 105 (4), 657–699.

Comanor, William S. and Schankerman, Mark A., 1976. Identical Bids and Cartel Behavior. Bell Journal of Economics, Spring, 7, 281-286.

Cook, P.W., Jr., 1963. Fact and Fancy on Identical Bids. Harvard Business Review, January-February, 41, 67-72.

Feinstein, J.S., Block, M.K., Nold, F.C., 1985. Asymmetric Information and Collusive Behavior in Auction Markets. American Economic Review, American Economic Association, vol. 75(3), pages 441-60, June.

Hendricks, K., Porter, R., 1988. An Empirical Study of An Auction with Asymmetric Information. American Economic Review, 78: 865-883.

McAfee, R.P., McMillan, J., 1992. Bidding rings. American Economic Review 82 (3), 579–599.

Mund, V.A., 1960. Identical Bid Prices. Journal of Political Economy, April 1960, 68, 150-69.

Padhi, S.S., Mohaparta, P.K.J., 2011. Detection of collusion in government procurement auctions. Journal of Purchasing and Supply Management, doi:10.1016/j.pursup.2011.03.001

Pesendorfer, M., 2000. A Study of Collusion in First-Price Auctions. Review of Economic Studies, Blackwell Publishing, vol. 67(3), pages 381-411, July.

Porter, R.H., Zona J.D., 1993. Detecting of bid rigging in procurement auctions. Journal of Political Economy 101 (3), 518-538.

Porter, R.H., Zona J.D., 1999. Ohio School Milk Markets: An Analysis of Bidding. Journal of Economics, The RAND Corporation, vol. 30(2), pages 263-288, Summer.